This short document describes the relation between the WEIGHT, SIGMA, SIGMA_SPECTRUM and WEIGHT_SPECTRUM columns of the MS and how various operations affect the values.

## Filler / Initialization

| Current Implementation | | Current Definition | | Proposed Definition | | |
|---|---|---|---|---|---|---|
| **Sigma** | **Weight** | **Sigma** | **Weight** | **Sigma** | **Weight** | **Weight Spectrum** |
| **1** | 1 | $\frac{1}{\sqrt{2\Delta\upsilon\Delta t}}$ | $2N_{chan}\Delta\upsilon\Delta t$ | $\frac{1}{\sqrt{2\Delta\upsilon\Delta t}}$ | $2\Delta\upsilon\Delta t$ | $2\Delta\upsilon\Delta t$ |

The weight column after initialization should be proportional to $1/\sigma^2$. The current definition of the weight column specifies that it contains the weight of the full spectral window and thus there is a factor of $N_{Chan}$ in the current implementation, which is not uniformly respected in the implementation. The proposed definition changes this and maintains in the WEIGHT column the per channel weight.

The current implementation inconsistently fills the WEIGHT Column with the value 1, the correct value should be $N_{Chan}$. This initial inconsistency increases the confusion regarding the meaning of the WEIGHT column rise to many of the problems we see in the current implementation (I think). Note that the factor of $\sqrt{2}$ in the definition of SIGMA is for cross-correlations only. For auto-correlation data the correct value is $(\Delta\upsilon\Delta t)^{-\frac{1}{2}}$.

## Effects of Calibration on WEIGHT and WEIGHT_SPECTRUM

The applycal (and pre-apply tables in calibration solves) all take the value contained in the SIGMA column as the basis for the derivation of the weight column. Tasks such as split and mstransform which produce a new MS and place modified values into the DATA column should ALWAYS update the SIGMA column to be consistent.

Throughout this section we describe the per antenna multiplicative factor to be applied to correctly calculate the weight value. So for an operation with weight scaling factor $\omega_i$ where the subscript denotes the $i^{th}$ antenna the WEIGHT column for a visibility on baseline between antenna$_i$ and antenna$_j$ with the value SIGMA given by $\sigma_{ij}$ is

$$W_{i,j} = \frac{\omega_i \omega_j}{\sigma_{ij}^2}$$

For autocorrelation observations, $\omega_i = \omega_j$ and the correct scale factor is $\omega_i^2$.

In the following tables we use the subscript $k$ to denote channel and the notation $\langle \quad \rangle_k$ to denote an average across channel.

### System Temperature

| Current Implementation | Final Implementation | |
|---|---|---|
| **Weight** | **Weight** | **Weight Spectrum** |
| $1 \Big/ \langle \sqrt{T_k^{Sys}} \rangle_k^2$ | $1 \Big/ \langle T_k^{Sys} \rangle_k$ | $1 \Big/ T_k^{Sys}$ |

This is the only calibration operation that will create a weight spectrum in the current implementation.

### Gain Calibration

| Current Implementation | Final Implementation |
|---|---|
| **Weight** | **Weight** |
| $\|\|\boldsymbol{G}\|\|^2$ | $\|\|G\|\|^2$ |

This one is simple, but this is the point at which the weights for the 7-m and 12-m are differentiated, since G is the only place in the calibration where $A_{eff}$ is accounted for.

### Bandpass

| Current Implementation | Final Implementation |
|---|---|
| **Weight** | **Weight** |
| $1 \Big/ \langle \|\|\boldsymbol{B}\|\|^{-1} \rangle_k^2$ | $\langle \|\|B\|\|^2 \rangle_k$ |

Note that for a normalized bandpass this implies that the weights are unchanged. For un-normalized bandpass tables, the result is an equal change to the weights of all channels.

### Time and Channel Averaging

Time and channel averaging data simply means summing the weights of the data in the sum. With the current definition of weight (weight of the entire SPW) this is a

well-defined operation, as the total weight of the new spectral window can be correctly calculated.

In general these operations produce channel dependent weights and should produce a SIGMA_SPECTRUM in the new MS, and an associated WEIGHT_SPECTRUM. In order to prevent unnecessary inflation of the MS size on disk we will preserve the option to produce a SIGMA and WEIGHT column, but the interpretation of this value must be understood to be the average weight of for all channels in the spectral window (which is effectively what happens now as well).

## Use of Weights in Imaging

The weights used in imaging are currently derived from the WEIGHT column, and have the following operations performed on them

- The weight of the parallel hand polarizations (RR & LL or XX & YY) are averaged and used for all imaging operation (including polarization imaging using the crosshand correlator products)
- Image weighting functions (Uniform, Briggs, etc) are applied.

## Implementation

### Phase 1:

An initial set of improvements will be made to support ALMA cycle 2 (June timeframe).

- We will implement a new tool method ms.initsigma which will have the following functionality:
  - Initialize the SIGMA column to $1/\sqrt{2\Delta\upsilon\Delta t}$ where $\Delta\upsilon$ is the EFFECTIVE_BW as given in the MS::SpectralWindow table and $\Delta t$ is EXPOSURE from the MS::MainTable.
    - An option will be preserved to initialize SIGMA to 1
  - Initialize the WEIGHT column to $\sigma^{-2}$

This tool can then be called in the SDM or other filler to correctly initialize the MS as part of the creation step. Note that any filtering applied online (such as Hann smoothing in the ALMA correlator) should be reflected in the EFFECTIVE_BW of the MS column to be correctly handled.

- The transition to the proposed weighting scheme and the switch to per channel weights stored in the WEIGHT column will be done.
- All calibration application operations will modified to conform to the Final Implementation described above
- MStransform and related tasks will be modified to respect these definitions and handle the weight column consistently.

### Phase 2:

A second phase of changes will be brought in to fully support weights consistently throughout the CASA package. These changes are

- Creation of WEIGHT_SPECTRUM when necessary (as well as option not to) in calibration tasks
- Optional creation of SIGMA_SPECTRUM (and handling in the code as necessary) when generating new MSs with heterogeneous channel weights.
- Creation / modification by StatWt task
- Modification of Imaging to
    - Use WEIGHT_SPECTRUM when available
    - Use average value of cross hands for imaging of polarization products