Thursday Lecture Series

July – Aug. 2011, Socorro

Lecture 3: Wide-field Imaging

July 28, 2011

S. Bhatnagar

Plan

- What do we mean by wide-field?
- Projection algorithms to correct for various wide-field effects
  - Relation with minor cycle algorithms
- Algorithms “unification scheme” :-)
  - Similarity between various wide-field algorithms
- Algorithms
  - For W-term correction
    - W-Projection, Multi facet Imaging
  - For PB corrections
    - A-Projection: Low and high frequency
  - AW-Projection at low frequency bands
- Connection with Mosaicking:
  - Generalization of single pointing

What do we call Wide-field?

- Imaging that requires invoking any of the following:
  - Corrections for non co-planar baseline effects
  - Corrections for the rotational asymmetry of the PB
    - Imaging beyond 50% point, mosaicking
  - Corrections for the frequency or polarization dependent effects
    - PB, ionosphere/atmosphere
- Noise limited imaging at “low” bands (L, S and probably C Band)
  - Because of the radio brightness distribution
- Noise limited imaging of structure comparable to the PB beam-width

$$I_{Continuum} = \int PB(\nu)\left[I_o(\nu/\nu_o)^{\alpha(\nu)}\right] d\nu\, dt = \int I_o(\nu/\nu_o)^{\alpha_{pb}(\nu,t)+\alpha(\nu)} d\nu\, dt$$

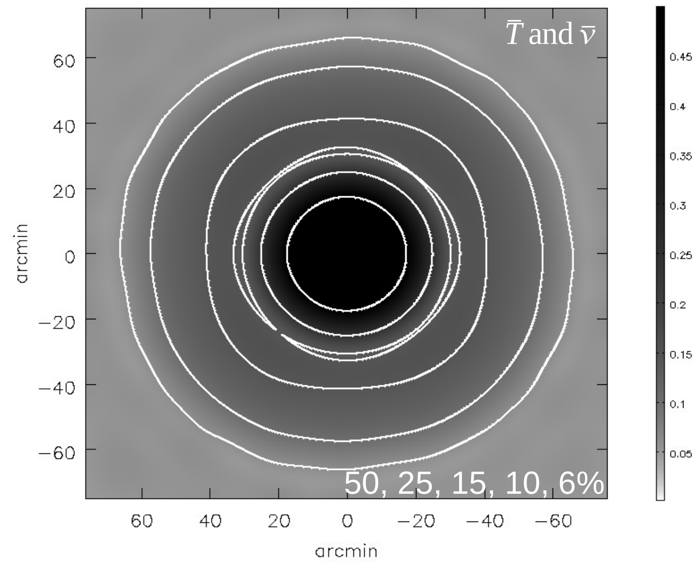
- Mosaicking
  - By definition, imaging on scales larger than the PB beam-width

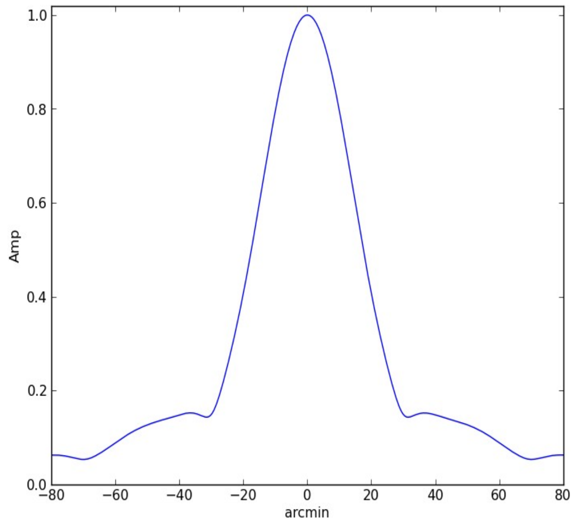
Why wide-field?

- Primarily due to improved continuum sensitivity
- E.g. a 1% PSF side lobe due to a source away from the center is now significantly above continuum thermal noise limit
  - This is a largely independent of the total integration time
- Due to large bandwidth, EVLA is sensitive father out in the FoV
- E.g. @L-Band, PB gain ~1 deg. away can be up to 10%
  - In the EVLA sensitivity pattern, VLA sensitivity is achieved at the location of VLA-null!
  - No null in the EVLA sensitivity pattern

Wide-field Issues

- For the same integration time, EVLA is sensitive to emission farther out



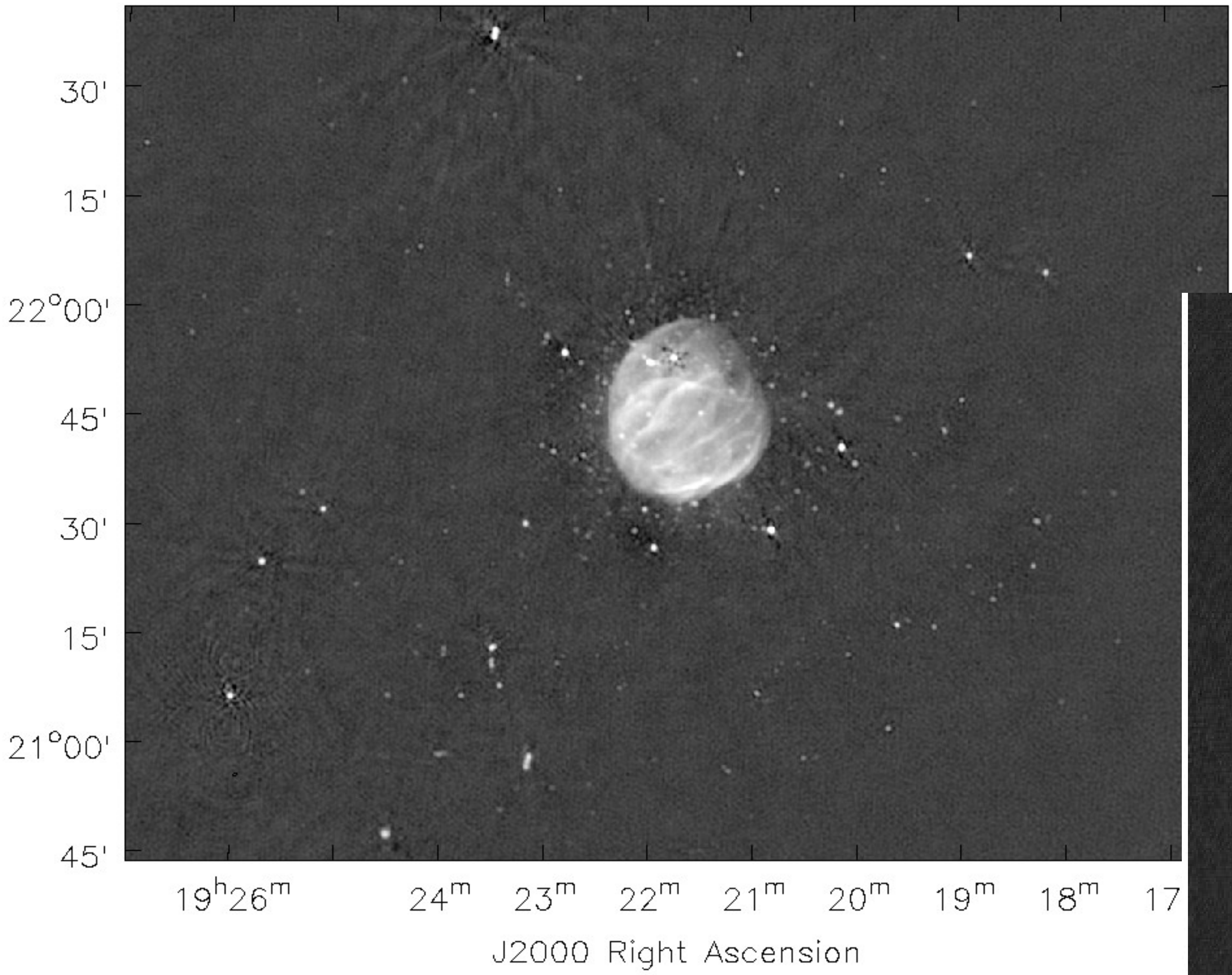


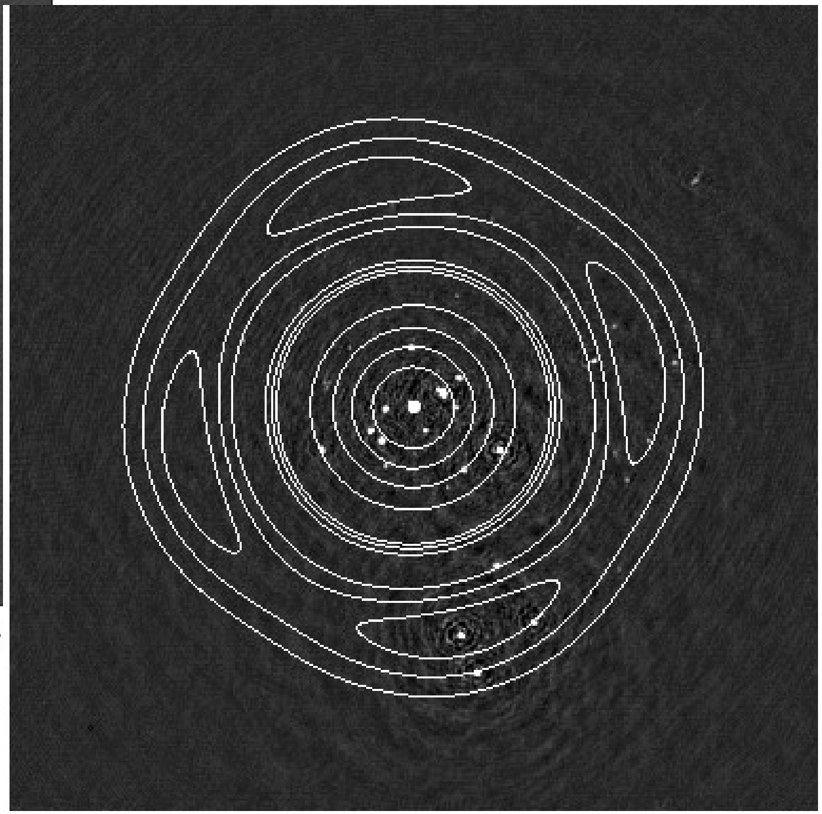
- Error at the center of the image due to a source at a distance R

$$\Delta S = S(R) \times PB(R) \times PSF(R)$$

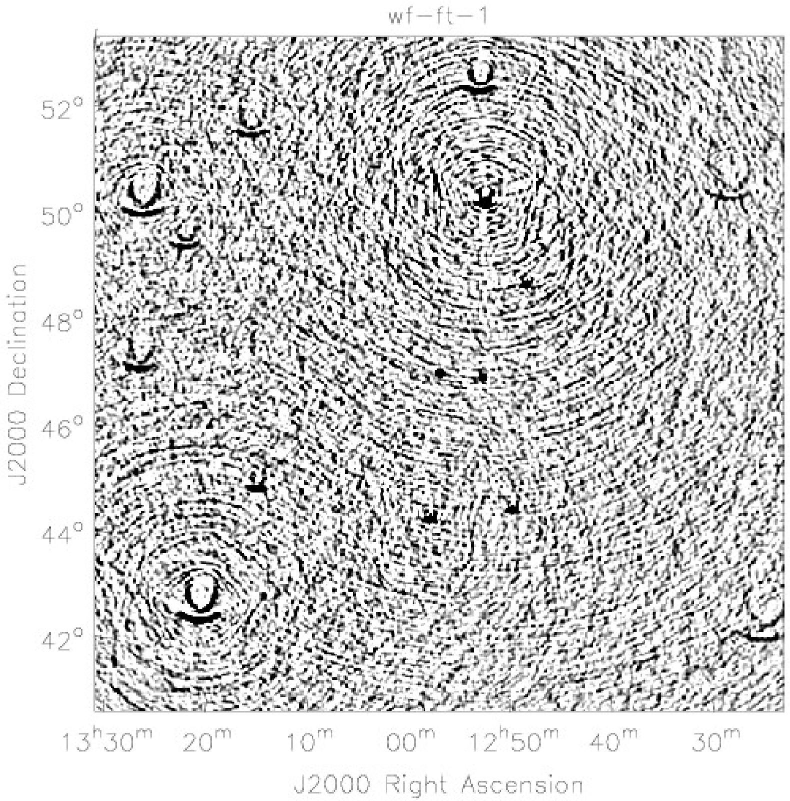
 - R = 1°, S(R)=1Jy, $\Delta S = 1 mJy - 100\,\mu Jy$

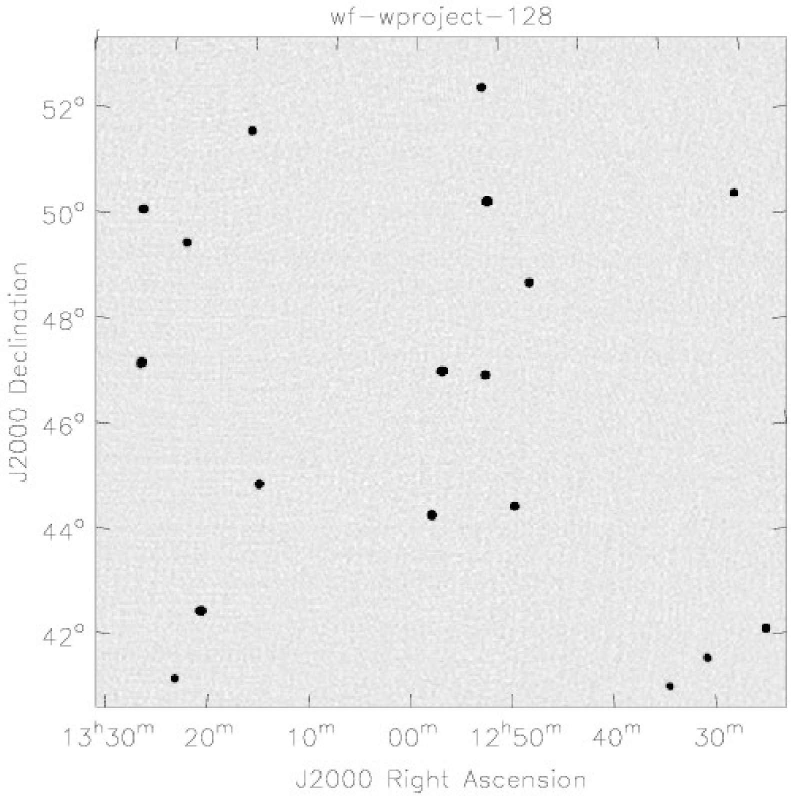
Wide-field Issues





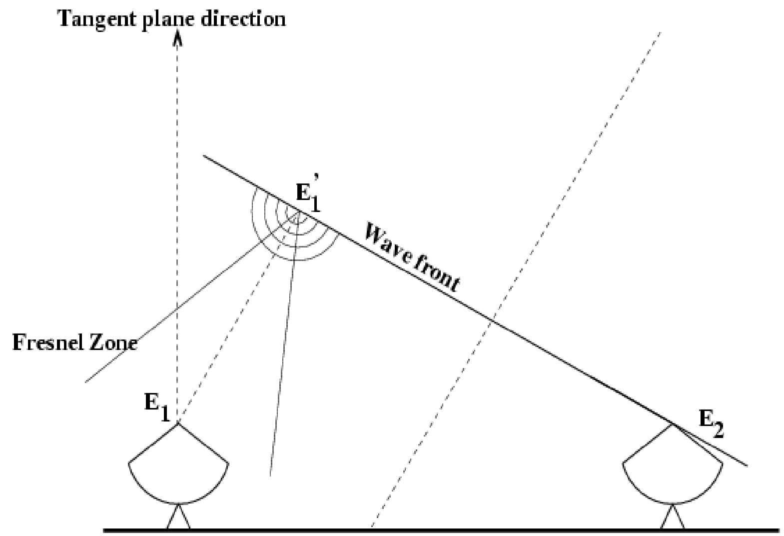
Effects of the W-Term





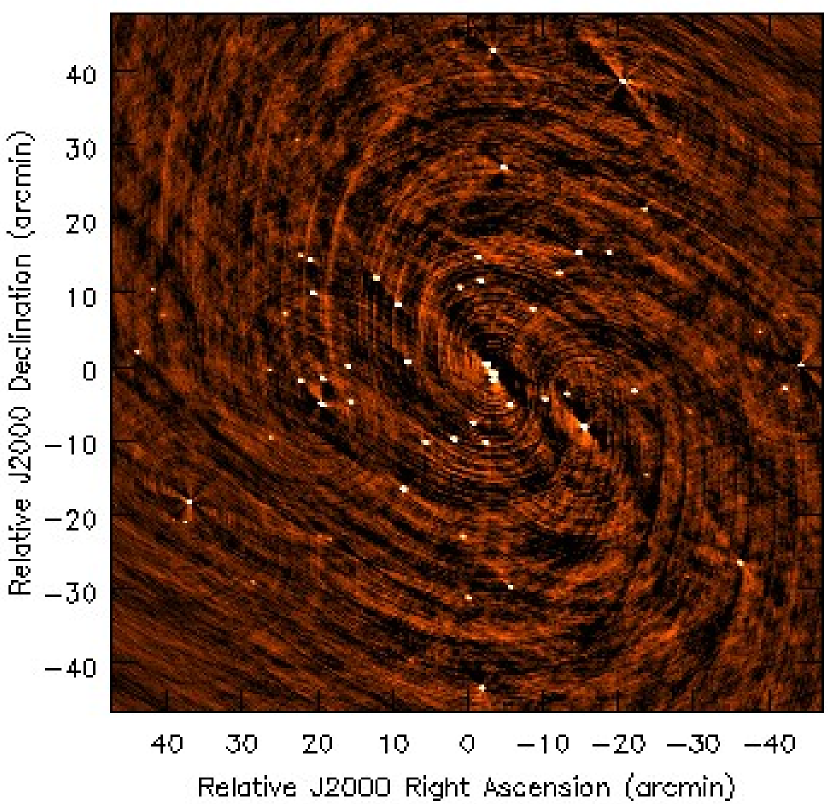
Non co-planar baseline: The W-term

- 2D FT approximation of the Measurement Equation breaks down
  - $\frac{\lambda}{B_{max}} \leq \theta_f^2 \qquad \theta_f = \textit{Angular distance from the phase center}$

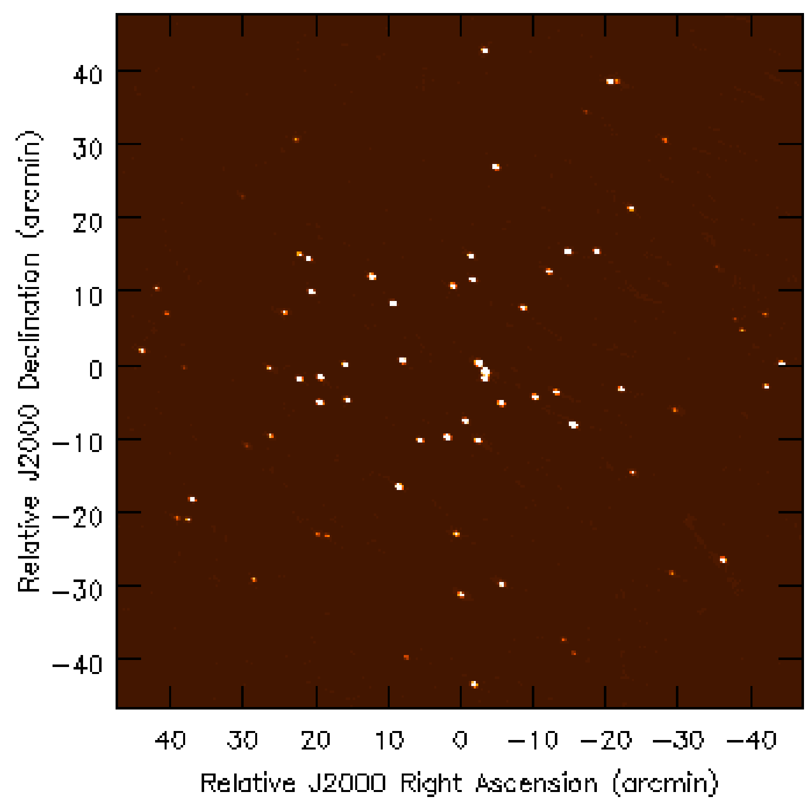


- We measure: $V_{12}^o = \langle \boldsymbol{E}_{\boldsymbol{1}}'(u, v, w \neq 0)\, \boldsymbol{E}_{\boldsymbol{2}}^*(0,0,0) \rangle$
- We interpret it as: $V_{12} = \langle \boldsymbol{E}_{\boldsymbol{1}}(u, v, w = 0)\, \boldsymbol{E}_{\boldsymbol{2}}^*(0,0,0) \rangle$
- We should interpret E_1 as [E_1' x Fresnel Propagator]

PB Effects



40
30
20
10
0
−10
−20
−30
−40
Relative J2000 Declination (arcmin)
40 30 20 10 0 −10 −20 −30 −40
Relative J2000 Right Ascension (arcmin)

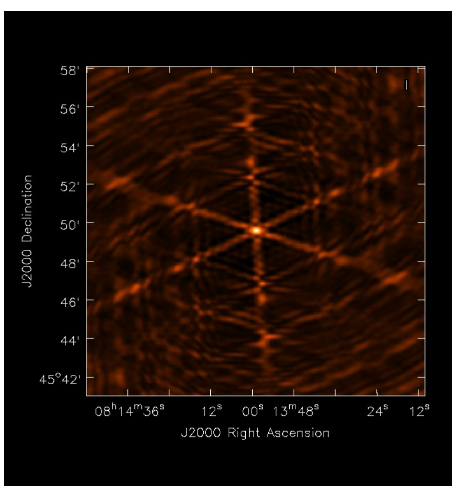


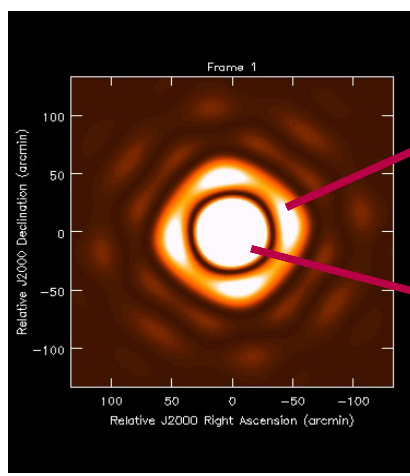
40
30
20
10
0
−10
−20
−30
−40
Relative J2000 Declination (arcmin)
40 30 20 10 0 −10 −20 −30 −40
Relative J2000 Right Ascension (arcmin)

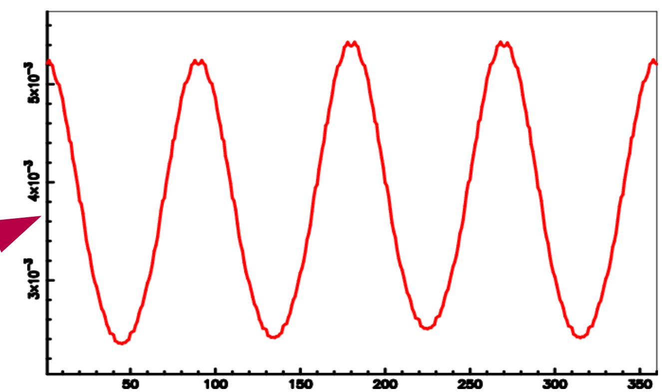
PB Effects: Rotation asymmetry

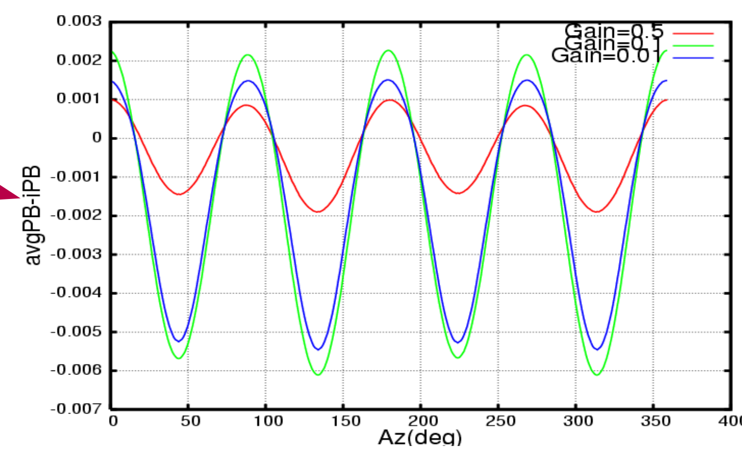
- Only average quantities available in the image domain
- Asymmetric PB rotation leads to time and direction dependent gains

$$\Delta I^R = \sum_{\psi} \left[PSF(\psi) - avgPSF \right] * \left[\left(PB(\psi) - avgPB \right) I^o \right]$$





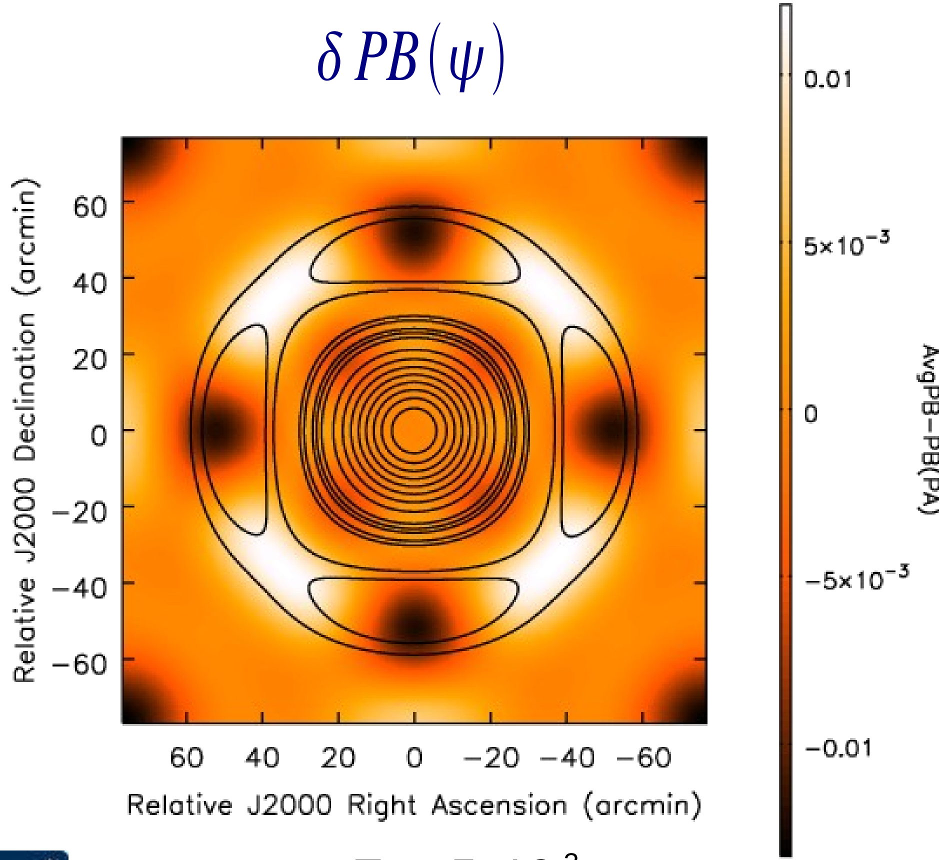




PB Effects: Error Propagation

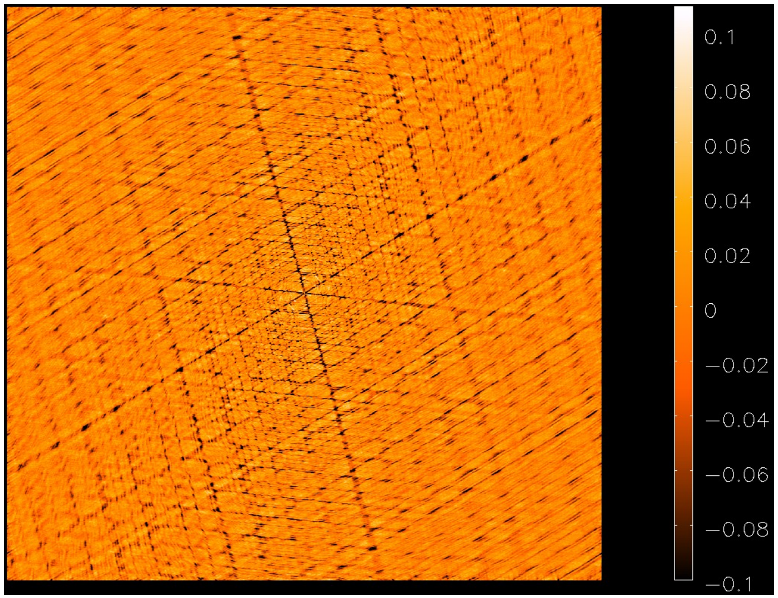
$$\Delta I^R = \sum_{\psi} \delta PSF(\psi) * [\delta PB(\psi) I^o]$$

$\delta PB(\psi)$



E.g. $5\text{x}10^{-3}$

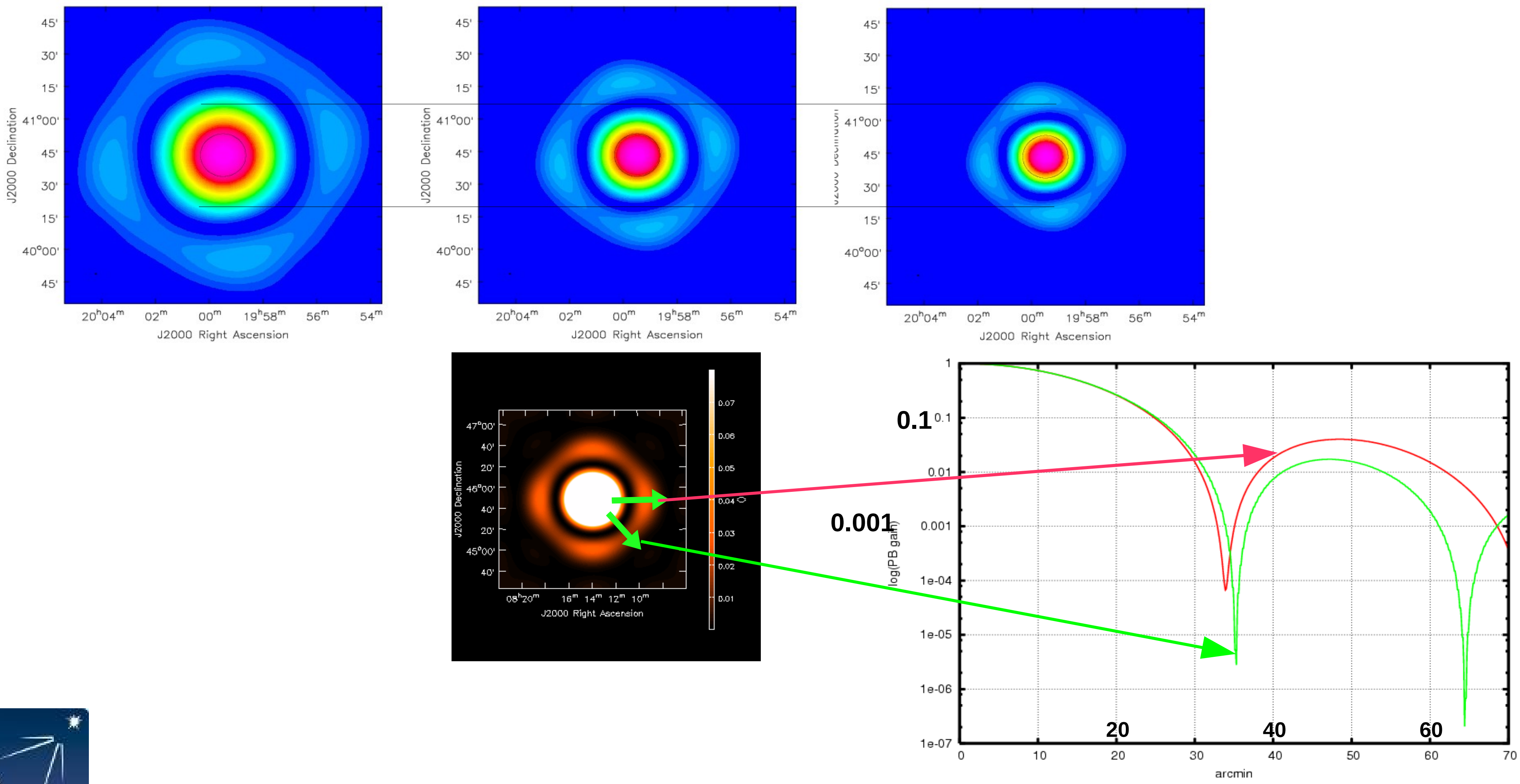
$\delta PSF(\psi)$



E.g. $2\text{x}10^{-2}$

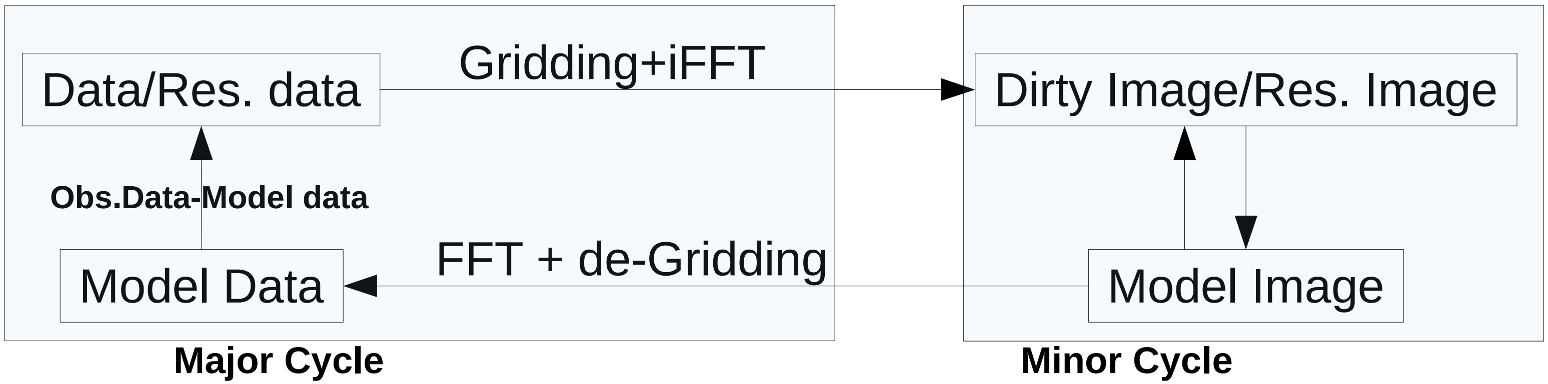
Frequency dependence of the PB

- Assume linear scaling with the frequency



Algorithms: CS Clean recap

- Compute residual using original data
  - Needs Gridding and de-Gridding during major-cycle iterations



- Most commonly used algorithm
- Every major cycle access the entire data base
  - Significant increase in I/O and computing load
- Assumes, co-planar, time- and freq-independent Measurement Equation
- Cannot account for wide-field wide-band and time variability issues

Deconvolution as ChiSq Minimization

- $V^M = A\, I^M + A\, N$ $\qquad\qquad$ $V_{ij} = deGrid_{ij}\, FT(I)$
- Non-linear solver, to solve for the Model Image
  - Compute residuals: $\quad$ **V^{Obs} - AI^M** $\quad$ (data domain)

    I^d - BI^M $\quad$ (image domain)
  - Make Residual Image **I^{res}**
  - Find update direction: Steepest Descent Algorithm

    $$I^c = max\left(-2[I^{Res}]\frac{\partial \chi^2}{\partial\, Param}\right)$$
  - Update model: $\quad I_i^M = T(I_{I-1}^M) \qquad for\, our\, discussions\, this\, is = I_{i-1}^M + \alpha * I_i^c$
  - Since Major Cycle does model subtraction without averaging, variable terms can be included in that step

Major Cycle
(always expensive)

Minor Cycle
(can be expensive)

Algorithms "unification scheme"

- Incorporates direction dependent effects as part of the gridding function
  - ME: $V_{ij} = A_{ij} I^o + N_{ij}$
  - Construct D, such that $\frac{D_{ij}^T A_{ij}}{D_{ij}^T D_{ij}} \approx 1$
  - Compute residuals (major cycle): D_{ij} for forward and D_{ij}^T for reverse transform
- W- and A-Projection construct ***D*** differently
  - A-Projection has additional normalization issues:
    - Flat-noise vs. flat-sky normalization
- Mosaicking: (more in K. Golap's lecture later)

$$I^{Mosaic} = \sum_k I(l_o - l_k)$$

$$\text{Use } D_{ij} e^{\iota[(l_o - l_k).u_{ij}]} \text{ where } D_{ij} \text{ can be } A_{ij}, W, \text{or } A_{ij} * W$$

  - The Fourier transform shift theorem

Algorithms “unification scheme”

- “Single polarization” case: Single element of the Mueller Matrix

- Imaging

$$\boldsymbol{V}^{\boldsymbol{Grid}} = \boldsymbol{CF} * \boldsymbol{V}^{\boldsymbol{obs}}$$

$$\boldsymbol{I}' = FFT[V^{Grid}]$$

- Prediction (de-gridding):

$$\boldsymbol{V}^{\boldsymbol{Grid}} = FFT^{-1}[I^{M'}]$$

$$\boldsymbol{V}^{\boldsymbol{M}} = CF^T * V^{Grid}$$

- CF can be A-term, W-term, AW-term, wide- or narrow-band

Projection algorithms

- Direction-dependent ("image plane") effects as convolutional terms in the visibility domain
- ME entirely in the visibility domain: $\boldsymbol{V_{ij}^{O} = A_{ij} I^{M} = M_{ij} F I^{M} = M_{ij} \left[V^{M} \right]}$

$$\begin{bmatrix} V^{o}_{pp} \\ V^{o}_{pq} \\ V^{o}_{qp} \\ V^{o}_{qq} \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} \\ M_{21} & M_{22} & M_{23} & M_{24} \\ M_{31} & M_{32} & M_{33} & M_{34} \\ M_{41} & M_{42} & M_{43} & M_{44} \end{bmatrix} * \begin{bmatrix} V^{M}_{pp} \\ V^{M}_{pq} \\ V^{M}_{qp} \\ V^{M}_{qq} \end{bmatrix}$$

- Diagonal: "pure" poln. products
- Off-diagonal: Include poln. leakage

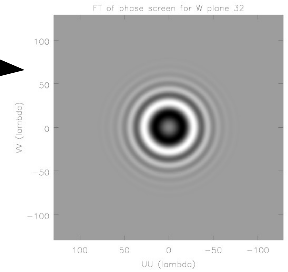
$$M_{pq} = J_{p,i} * J^{*}_{q,j}$$

- $\boldsymbol{V_{pp}^{O}} = M_{pp} * V_{pp}^{M} + M_{p\ \ p2q} * V_{pq}^{M} + M_{q\ \ p2q} * V_{qp}^{M} + M_{p2q\ \ p2q} * V_{qq}^{M}$
- Generalization of the direction-independent ME
  - Replace functions by complex numbers $\qquad M_{ij} = g_i g_j^{*}$
  - Replace convolution ('*') by complex product

W-Projection

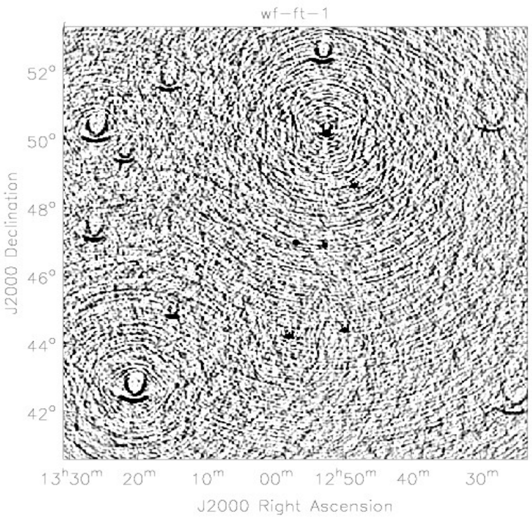
- W-Projection: (CASA Imager: ftmacine="wproject")

$$D = FT\left[e^{2\pi\iota\sqrt{w-1}}\right]$$



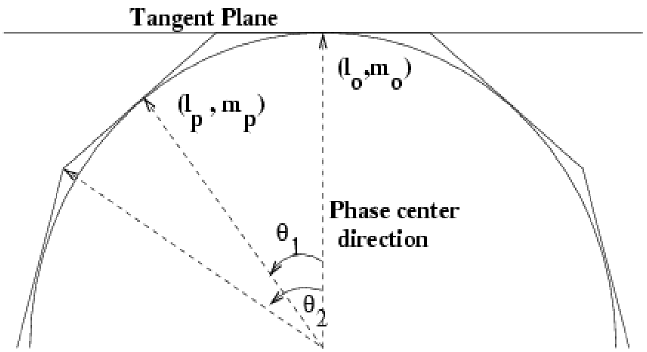
- $D^T A = 1$ by construction
  - Potentially fully corrects for the effects of the W-term
  - In practice, D is computed at a finite w-resolution, with interpolation in between

- D is non-hermitian
  - Post deconvolution correction is not possible
  - Same as: "corrections for antenna based phase errors cannot be corrected for post-deconvolution"



W-Projection + Multi-faceting

- Multi-facet imaging (CASA Imager: facets > 1)



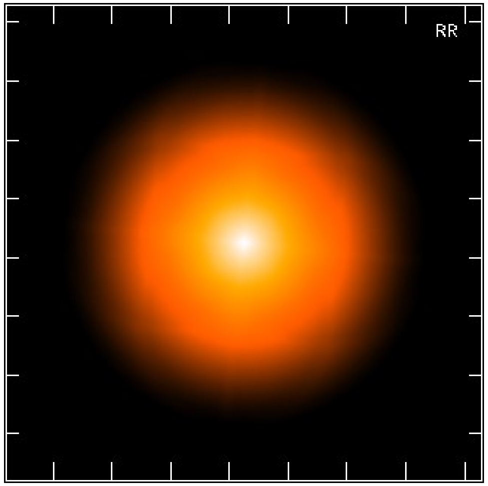
- Split the sky into multiple, smaller tangent-plane images
- A linear approximation of this image-plane operation is possible in the visibility plane: $\boldsymbol{I}(\boldsymbol{C}\,l) \rightarrow |det(\boldsymbol{C})|^{-1} \boldsymbol{V}(\boldsymbol{C}^{-1^T} u)$
  - Advantage: leads to a single combined image in the minor cycle
- Combination of W-Projection and Multi-facet imaging possible:
  - Reduces the no. of w-planes and number of facets

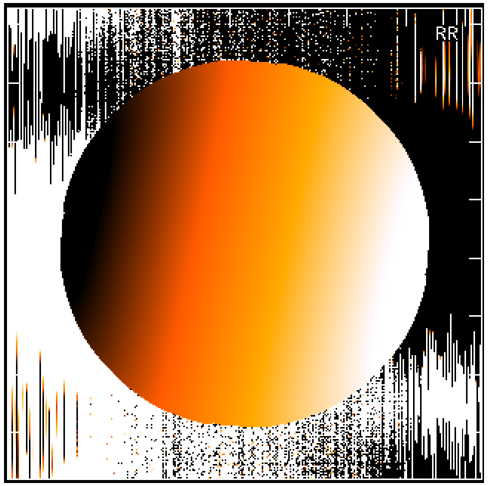
A-Projection

- A-Projection: $D =$ Auto-correlation of Aperture illumination function

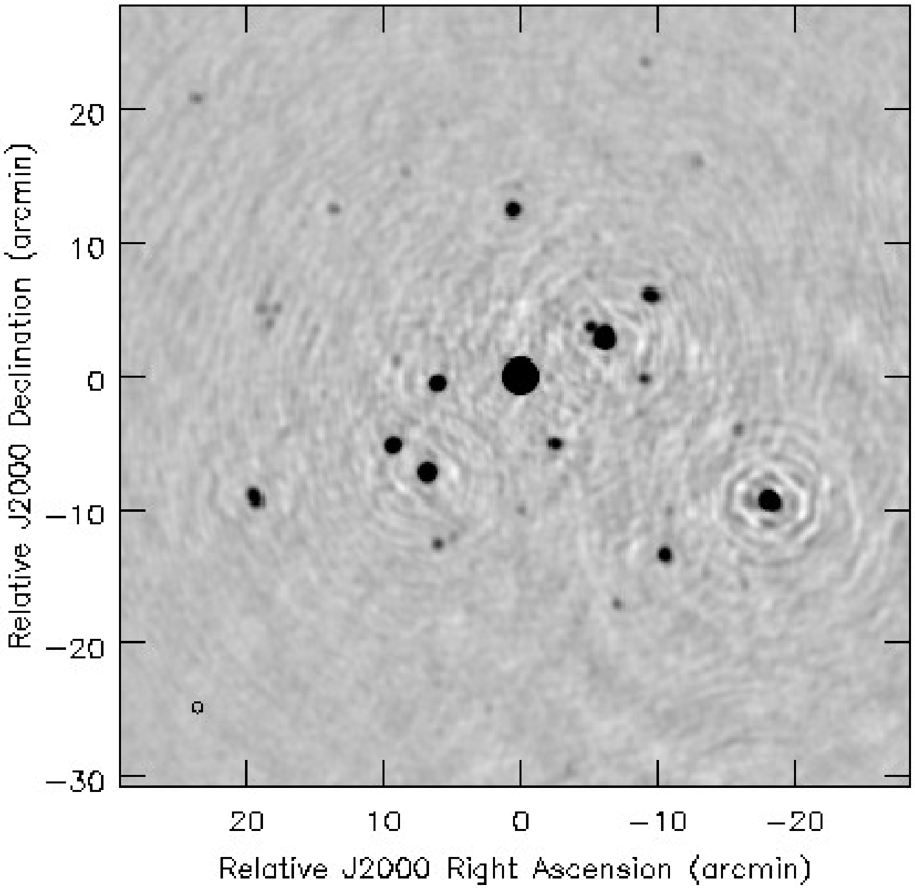
ftmacine="aproject

  - Function of time, frequency and polarization
- Since image is averaged over time and frequency, time- and frequency-dependence cannot be corrected for post-deconvolution
  - Same issue as non Hermitian nature of antenna based phase, W-term

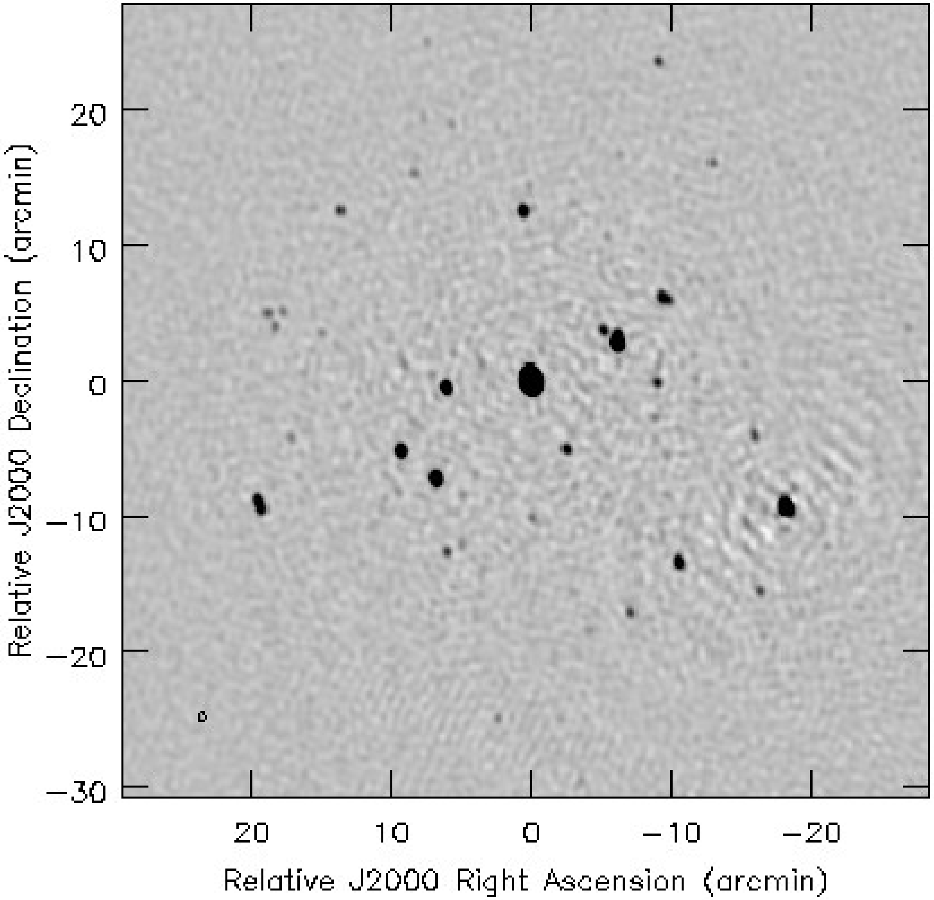




A-Projection: Before



A-Projection: After

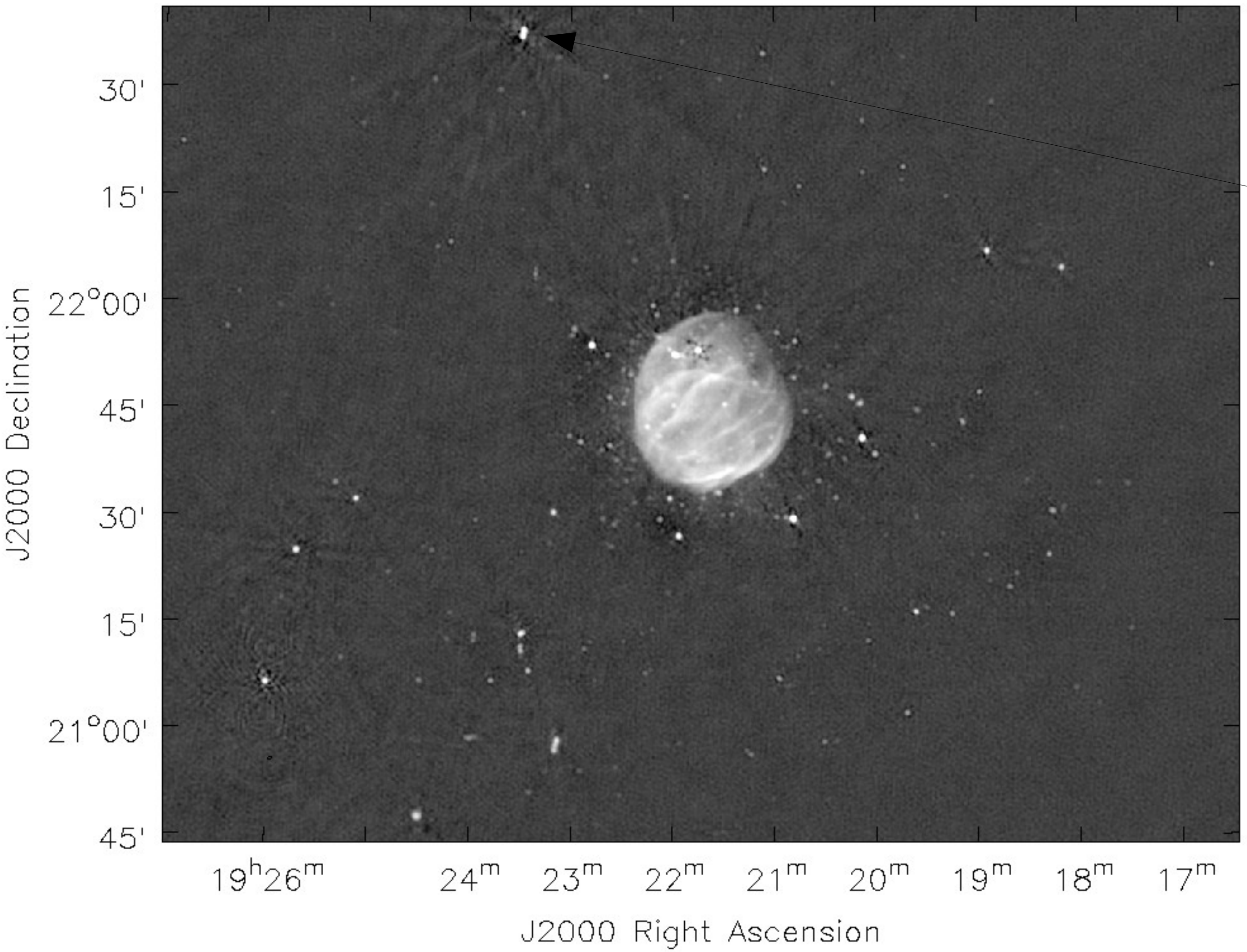


Imaging at high frequencies

- Definition: Frequency at which the array is co-planar for the required FoV
- To the first order, aperture illumination may linearly scale with frequency (or at least with in a certain range in frequency)
- Wavelength much smaller than the physical reflecting structures
  - Geometrical ray-tracing models might be sufficient
- Can be computed once per SPW, rotated in time, and scale in frequency during imaging
  - Significantly reduces memory foot print, at the cost of computing
  - Can be computed efficiently on GPUs

Imaging at low frequencies

- Definition: Frequency at which the array is non co-planar for the required FoV

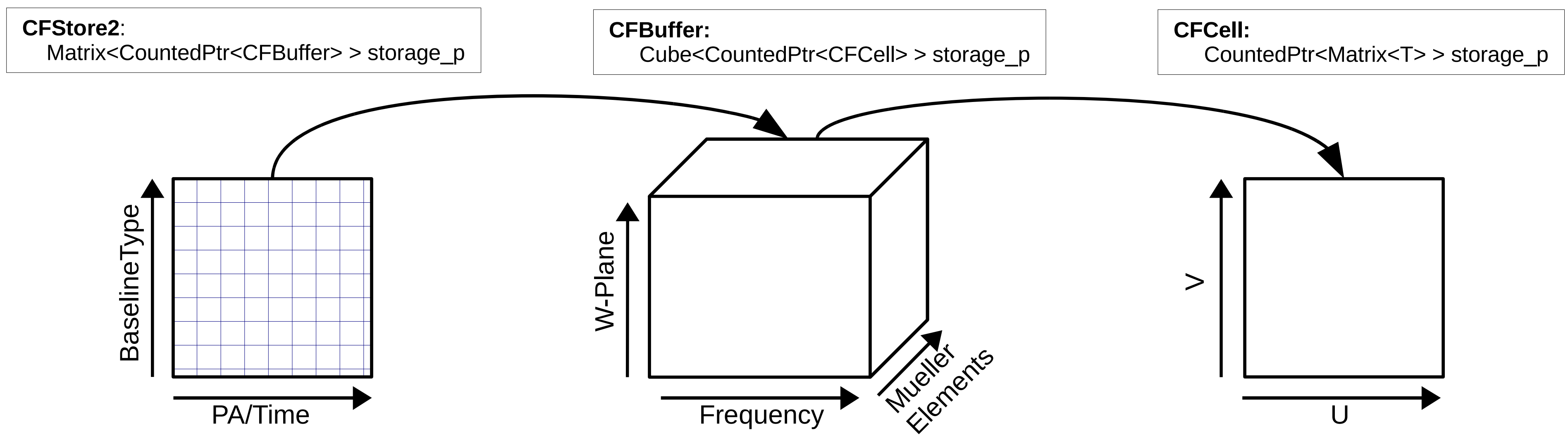


- PB variations with time
- Even D-array is non co-planar
- BW ~400 MHz

- Need:
  Wide-band AW-Projection

Wide-band AW-Projection

- $\mathbf{D}(\nu) \neq (A * W)(\nu/\nu_o)$
- Full-polarization case requires:

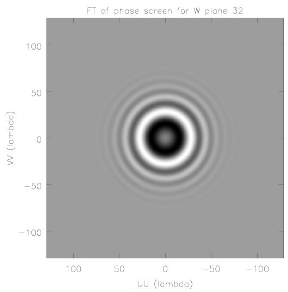


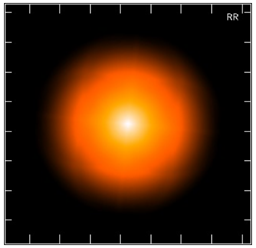
- Can be configured for optimal usage for:
  - High frequency: A-Projection, scaling with frequency
  - Low frequency: AW-Projection
  - Heterogeneous array

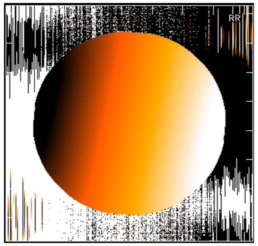
Physics of “unification”

- Physics of DD terms go into the construction of D
- Multiple DD terms become “convolution of convolution functions”

W <convolved> A







- E.g. form of the phase of A-term accounts for mosaicking, pointing corrections, etc.
- Wide-band, full-pol., low-freq. Mosaic can be done naturally
  - Complexity goes in the construction of the CFs
  - Rest of the imaging / calibration framework remains oblivious