A(perture)-to-Z(ernikes) Solver Methodology

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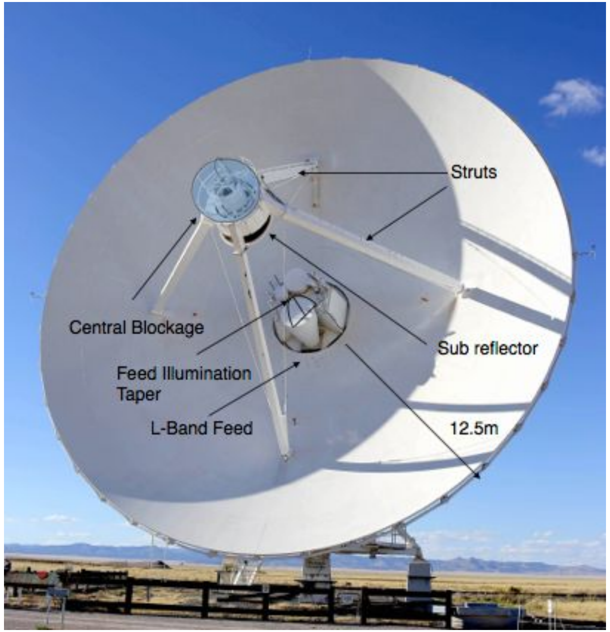
https://arxiv.org/abs/2107.10009

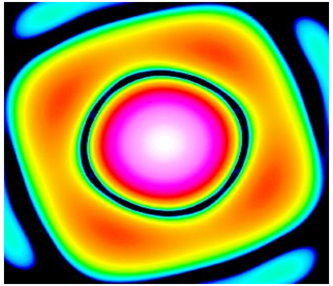
Outline

- **Primary Beams and Imaging.**
  - Antennas and primary beams
  - Measurement Equation
  - Image plane correction

- **Modelling antenna Aperture Illumination Patterns (AIP)**
  - Zernike Polynomial Basis
  - Modelling the AIP
  - Results

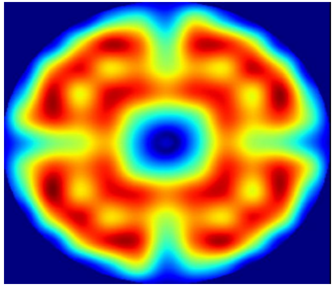
Antennas and Primary beams





PB

FT



AIP

Measurement Equation

$$V_{ij}^{Obs} = W_{ij}G_{ij}\int \mathbf{M}_{ij}(\mathbf{s})I_{ij}(\mathbf{s})e^{i\mathbf{b}_{ij}.\mathbf{s}}d\mathbf{s}$$

Direction independent Gains

Mueller Matrix

Sky Brightness Distribution

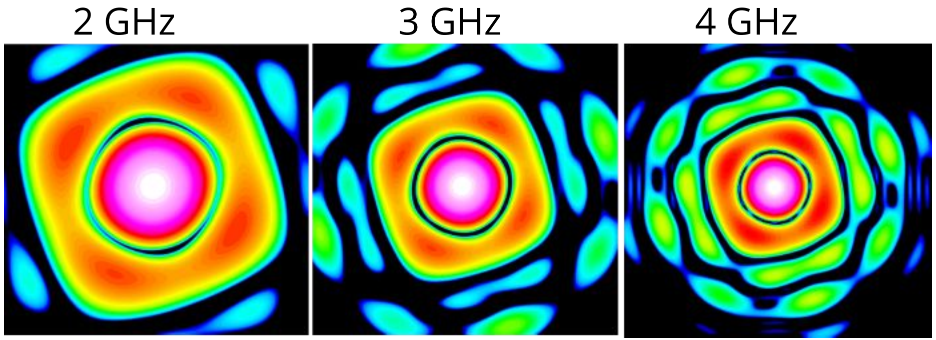
$$V_{ij}^{obs}(\mathbf{s},\nu,\theta_{PA}) = A_{ij}(\mathbf{s},\nu,\theta_{PA}) \circledast V_{ij}^{True}$$

Discretized version of the integral equation entirely in the Fourier Domain.

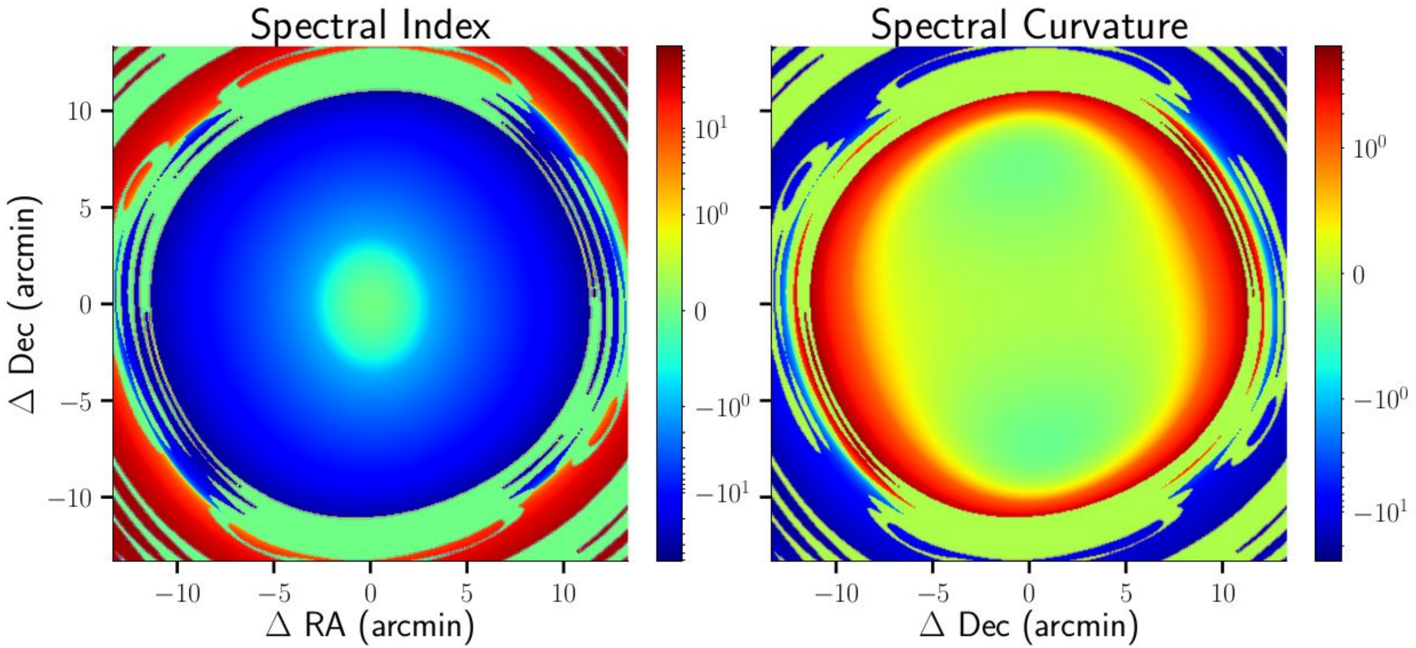
Wideband Primary Beams

- Primary beams are not azimuthally symmetric typically
  - Common approximation
  - Not suitable for HDR imaging
- Primary Beams vary with time for an Alt-Az mounted antenna

- Primary Beams are Chromatic



Spectral Behaviour of PB

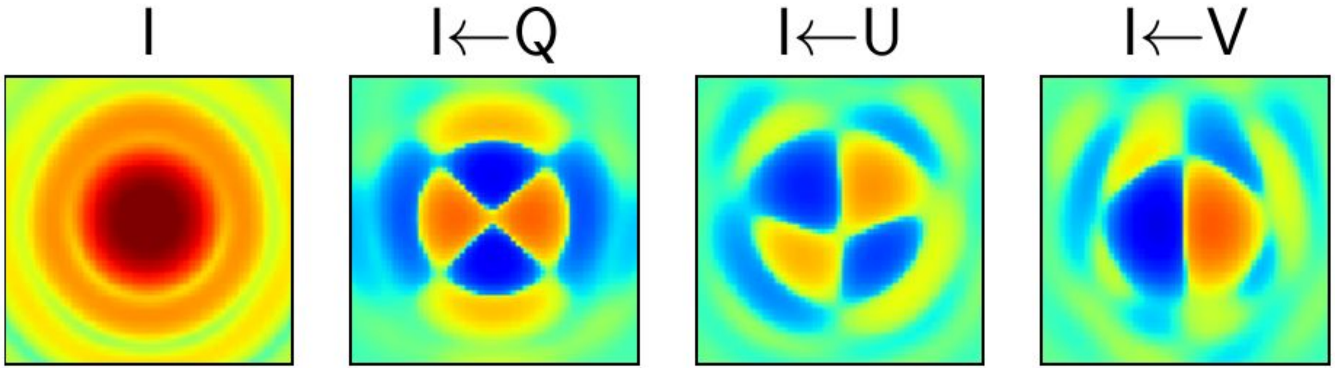


$$\alpha_{obs} = \alpha_{source} + \alpha_{beam}$$

Wideband Primary Beams

- Primary beams also encode the leakage of one stokes parameter into another.

MeerKAT L-Band

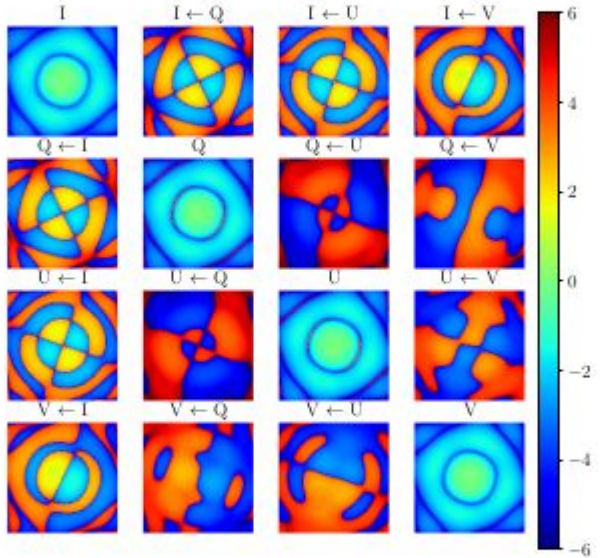


Sky Brightness & Mueller Matrices

$$\vec{V}^M_{ij} = \mathcal{F}\vec{I}^M = \mathcal{F}\sum_{i,\nu,t}\left(M_{ii}\cdot I_i^{sky} + \sum_{j,i\neq j} M_{ij} I_j^{sky}\right)$$

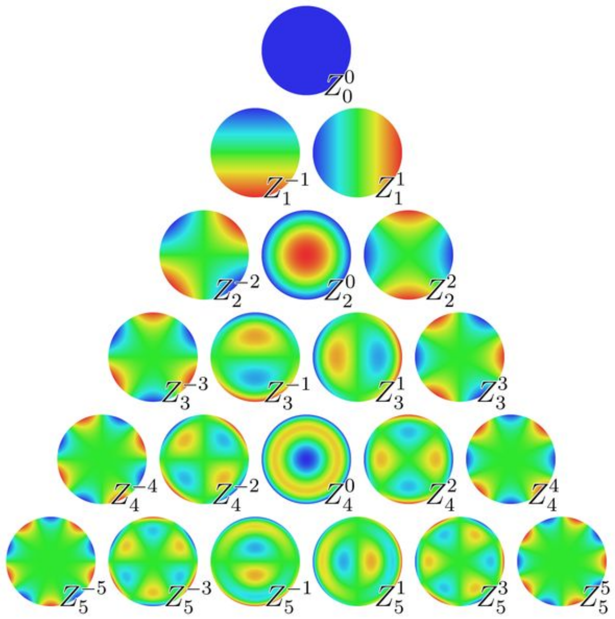
$$I^{\vec{M}} = \sum_k \begin{pmatrix} M^k_{\mathtt{II}}I + M^k_{\mathtt{IQ}}Q + M^k_{\mathtt{IU}}U + M^k_{\mathtt{IV}}V \\ M^k_{\mathtt{QI}}I + M^k_{\mathtt{QQ}}Q + M^k_{\mathtt{QU}}U + M^k_{\mathtt{QV}}V \\ M^k_{\mathtt{UI}}I + M^k_{\mathtt{UQ}}Q + M^k_{\mathtt{UU}}U + M^k_{\mathtt{UV}}V \\ M^k_{\mathtt{VI}}I + M^k_{\mathtt{VQ}}Q + M^k_{\mathtt{VU}}U + M^k_{\mathtt{VV}}V \end{pmatrix}$$

Where the index k can be time or frequency



Zernike Polynomials

- Originally defined to study ionosphere
- Defined on a unit circle
- Extensively used to model aperture illumination in optics
- Ties various polynomials to actual optical effects - astigmatism, coma etc.

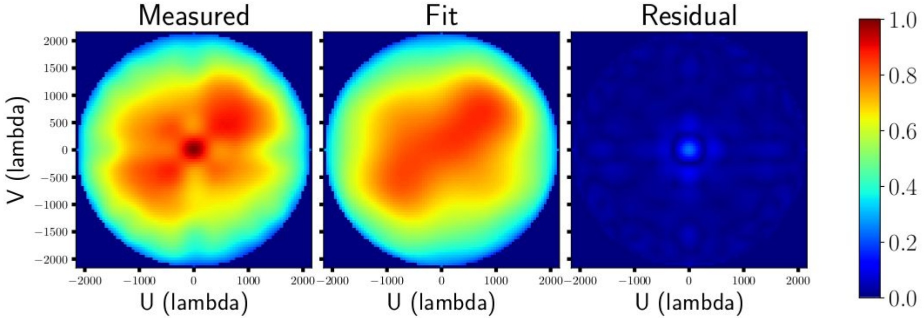


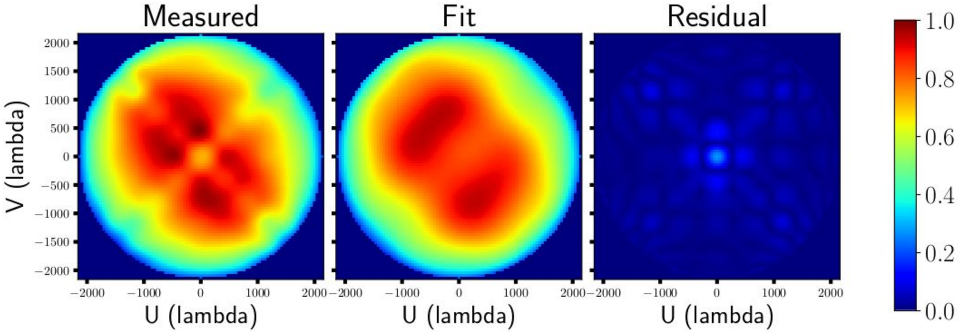
Aperture Modelling

- Use Zernike polynomials to model the complex aperture.

- Why aperture?
  - Natural domain to model optical effects that cause PB weirdness
  - Aperture size is fixed, determined by geometry of antenna dish.

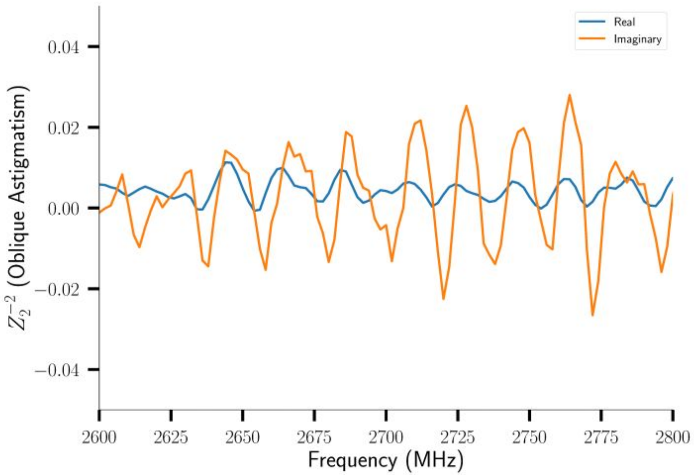
- Telescope agnostic - only requires holography measurements

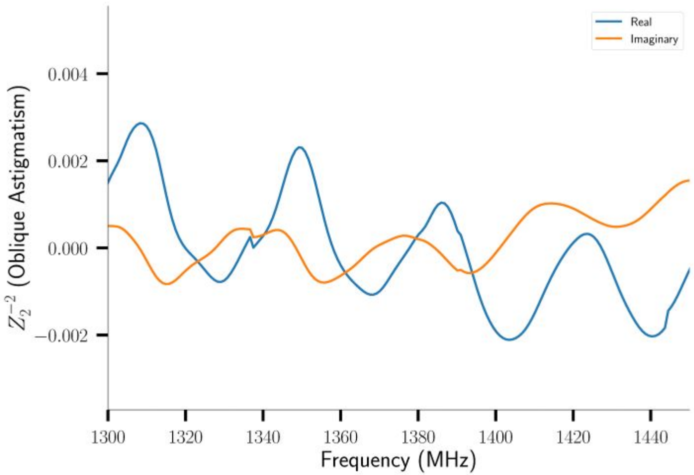




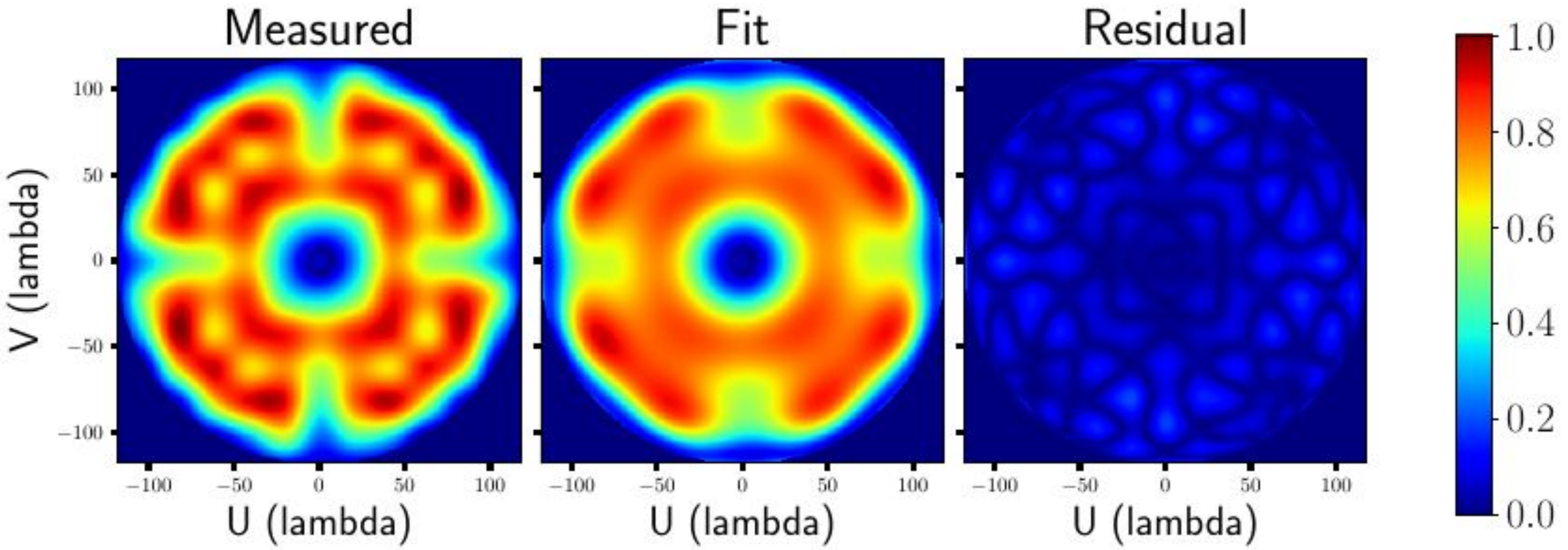
Aperture Modelling

- Any good modelling should find physical effects such as secondary reflection. EVLA - 17MHz and MeerKAT - 37MHz.

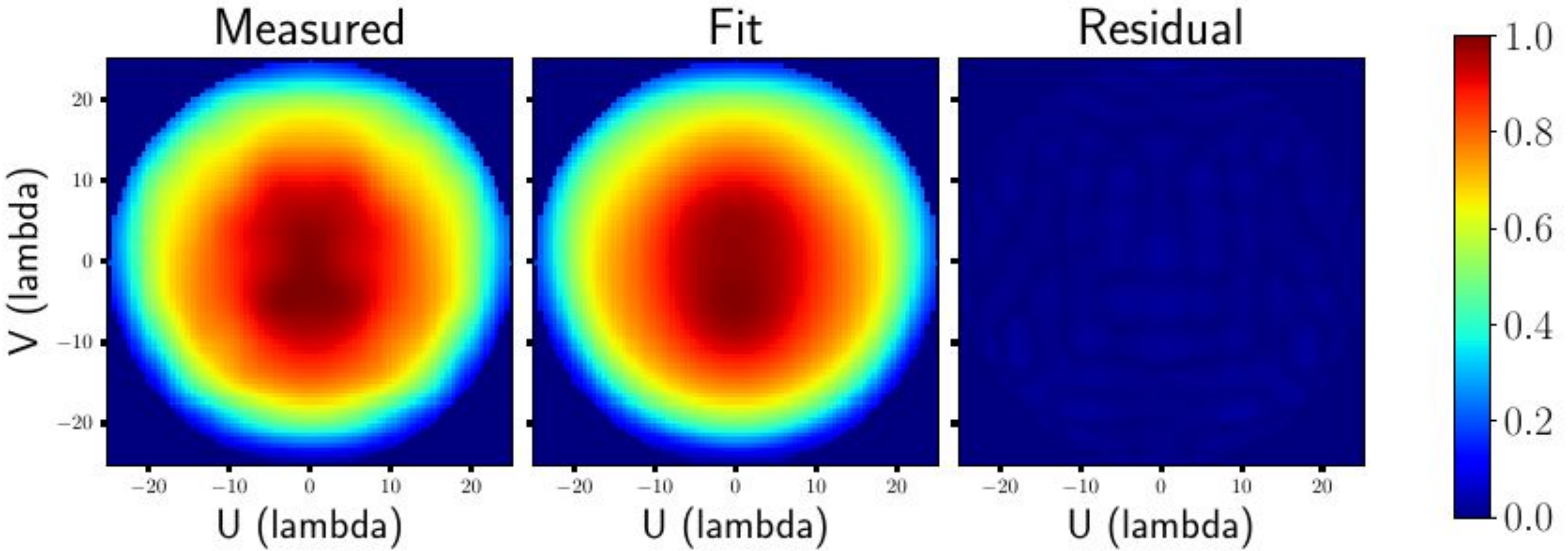




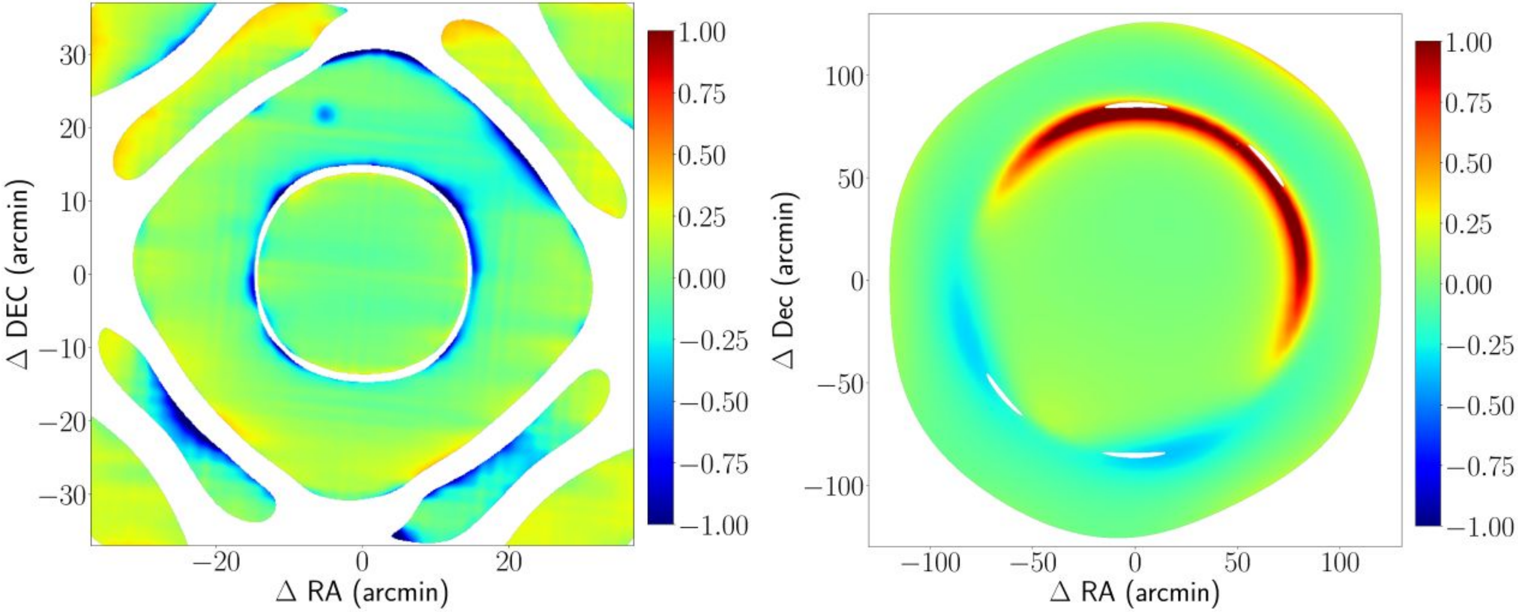
Aperture Models - EVLA S-Band



Aperture Models - MeerKAT L-Band



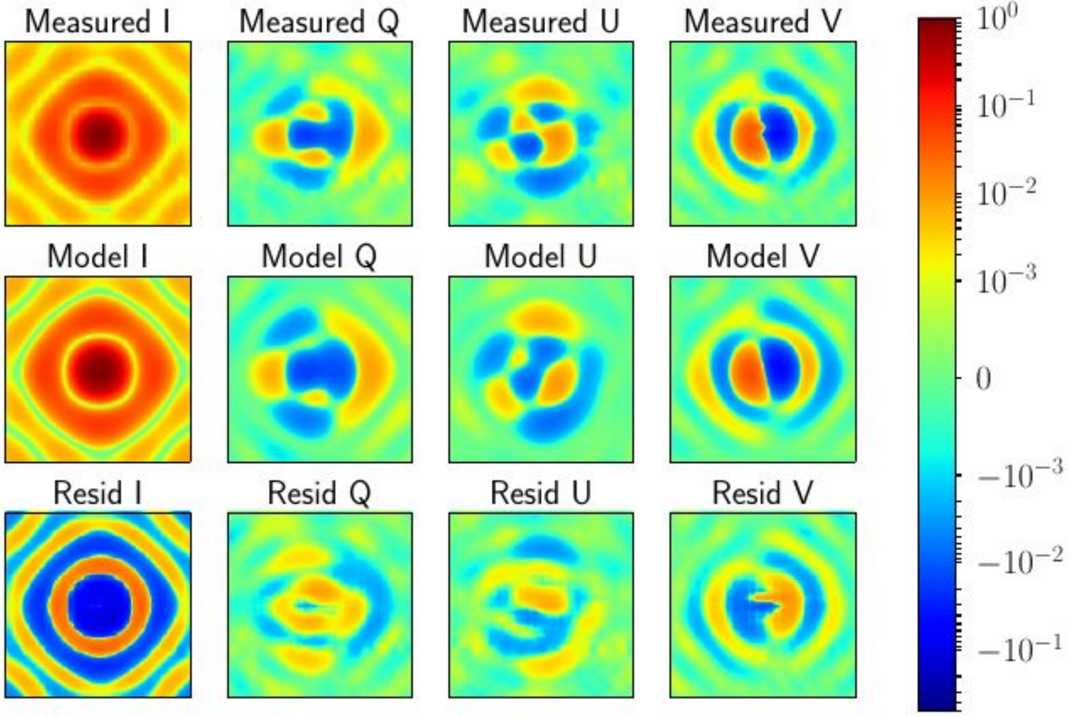
Fractional Residual PB



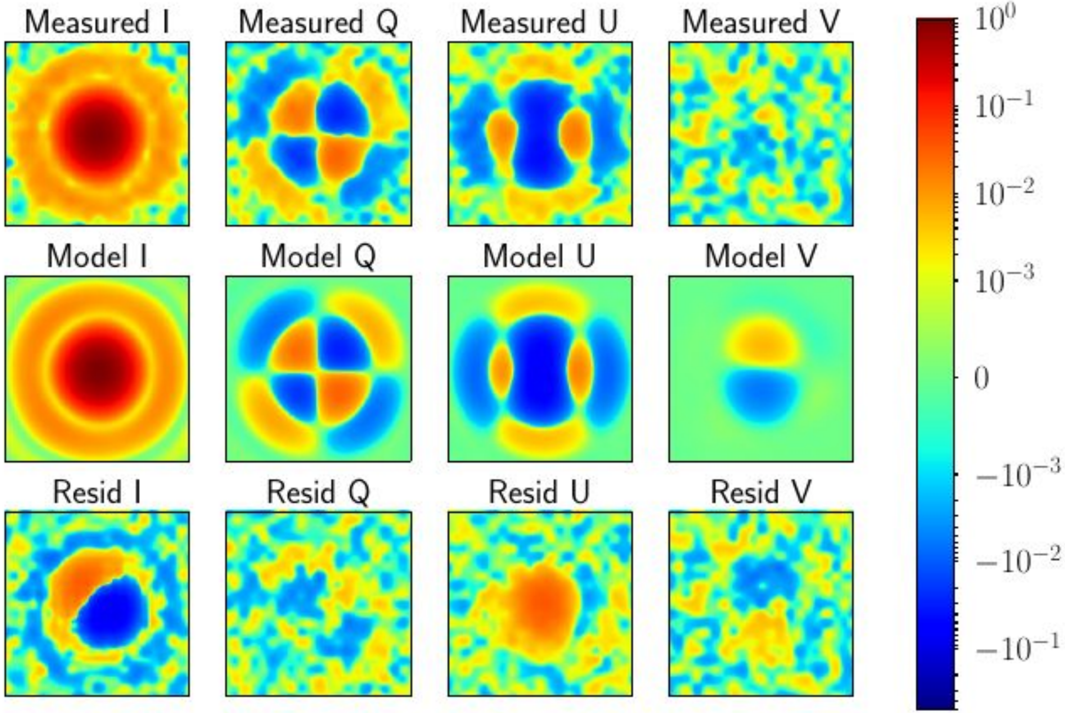
(a) EVLA

(b) MeerKAT

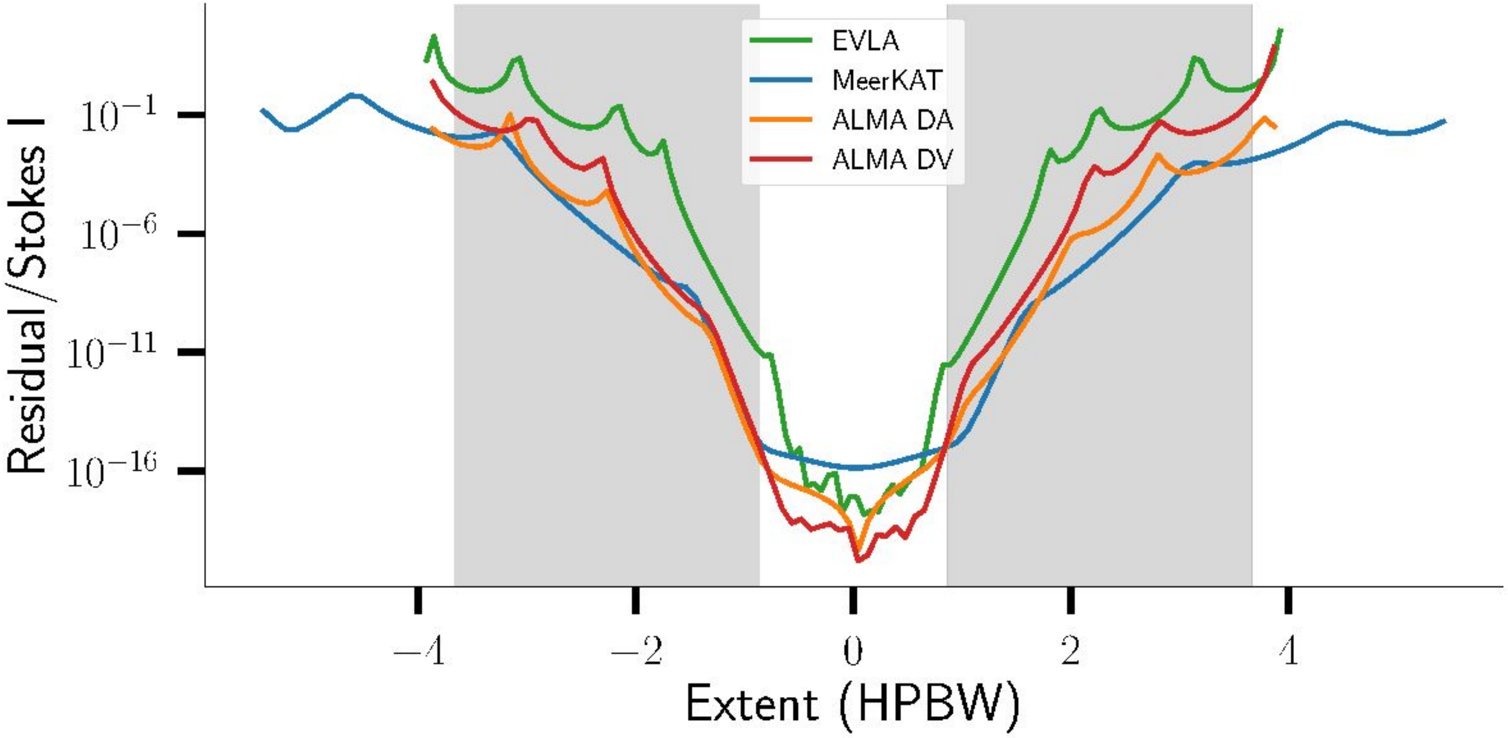
EVLA PB Models



ALMA PB Models



PB residuals

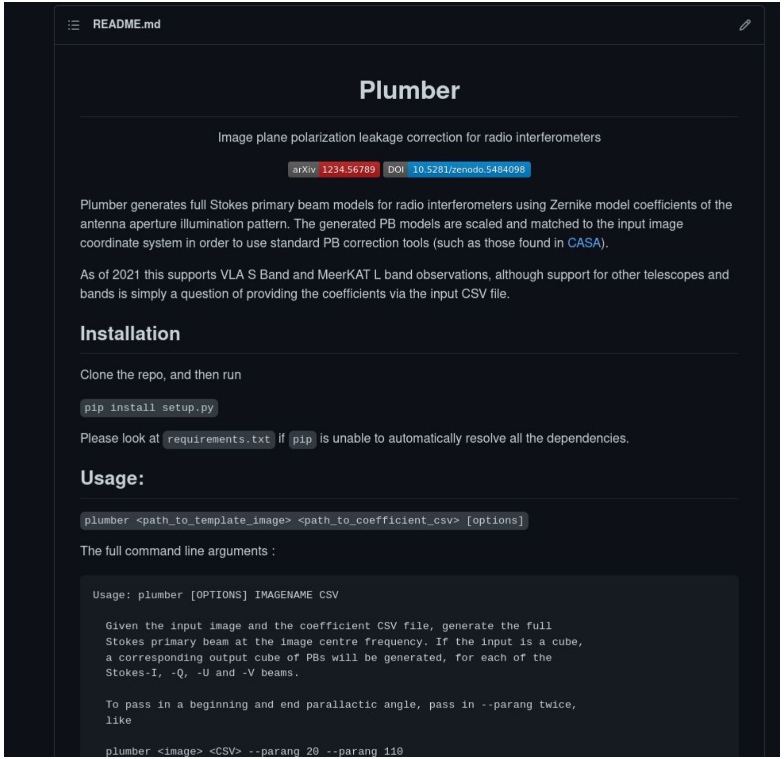


Summary

- Modelling PB and AIP are equivalent
- Zernike Polynomials are a natural basis for modelling complex antenna jones.
- We have successfully modelled PB's from three different telescopes, operating at widely different frequencies, across all polarizations.
- Our modelling allows for integration into projection like framework & image plane corrections
- Telescope agnostic - Limited only by availability of high quality Holography
- Image plane corrections are possible. Check out open source package on GitHub

Checkout our image plane correction package

https://github.com/ARDG-NRAO/plumber


README.md

Plumber

Image plane polarization leakage correction for radio interferometers

arXiv 1234.56789 DOI 10.5281/zenodo.5484098

Plumber generates full Stokes primary beam models for radio interferometers using Zernike model coefficients of the antenna aperture illumination pattern. The generated PB models are scaled and matched to the input image coordinate system in order to use standard PB correction tools (such as those found in CASA).

As of 2021 this supports VLA S Band and MeerKAT L band observations, although support for other telescopes and bands is simply a question of providing the coefficients via the input CSV file.

Installation

Clone the repo, and then run

`pip install setup.py`

Please look at `requirements.txt` if `pip` is unable to automatically resolve all the dependencies.

Usage:

`plumber <path_to_template_image> <path_to_coefficient_csv> [options]`

The full command line arguments :

```
Usage: plumber [OPTIONS] IMAGENAME CSV

  Given the input image and the coefficient CSV file, generate the full
  Stokes primary beam at the image centre frequency. If the input is a cube,
  a corresponding output cube of PBs will be generated, for each of the
  Stokes-I, -Q, -U and -V beams.

  To pass in a beginning and end parallactic angle, pass in --parang twice,
  like

  plumber <image> <CSV> --parang 20 --parang 110
```