Calibration of SKA1-LOW for EOR Imaging

http://www.timcornwellconsulting.com
SKA1-LOW

- 512 stations over 80km diameter region in Boolardy Station, Western Australia
- 256 antennas in each station
- 50MHz - 350 MHz
- Compact core for pulsars and EOR
- Spiral arms for imaging and ionospheric calibration
- Main source of calibration errors is the ionosphere through which the telescope images
- Can the ionosphere be calibrated for EOR imaging?
- Can we design array to optimise calibration?
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EOR imaging with SKA1-Low

- The pertinent L0 requirement is:
  - **SCI-REQ-18**: SKA1-LOW shall provide 50 dB brightness dynamic range at 300 arcsec spatial and 1 MHz spectral resolution to enable EoR imaging and power spectra generation at 50 – 200 MHz (HPSO 1 and 2).
Ionosphere

- TEC/Phase screen at about 300km
- Science requires 6 arcmin resolution image over 5 degree field of view
- Corresponds to 60km diameter region of the ionosphere
Overview

- Original motivation
  - Are there constraints on configuration arising from the need to calibrate for EOR?
  - To my surprise, constraints were not very important because the calibration was not possible even with the best array configuration

- SDP Memo 26
  - Residual phase error due to incomplete calibration of the ionosphere is $O(0.04)$ rad in 10s
  - Cannot reach 50dB in lifetime of telescope

- SDP Memo 27
  - Addressed various comments (on bandwidth, source counts), but found that the conclusions did not change
Effect of direction-dependent selfcalibration

GMRT    LOFAR+DIE    LOFAR+DDE
State of the art calibration?

- Self-cal on bright sources in the field
- Self-cal on clusters of sources
- Self-cal on facets within the field
- Self-cal on bright sources + phase screen fitting (SPAM)
- Not much work on analysis of limits
Piercing points

Estimate phase at pierce points then fit a screen
SKA1-Low images through an ionospheric phase screen of about 60 km diameter, stable on 10s timescale (at 100MHz)

To constrain the phase screen, use observations of foreground/background sources

To get 50dB in 5 years elapsed time (6 months actually on sky), integration over time yields 30dB so leaving a requirement for 20dB in a typical 10s snapshot

RMS in phase must be $10^{-2}$ radians in 10s across the field of view

Insufficient density of background sources

Full field dynamic range of 50 dB not possible within lifetime of telescope

Sufficiently restricted FOV is feasible
Number of sources available for pierce point analysis

- Ionosphere changes on 10s at 100MHz
- Not many sources in primary beam usable as calibrator sources in 10s/1MHz
- A given EOR field is only observable ~10% of the year
- Ionosphere goes through “bad periods”
Previous analytical work

- Cath Trott and Stefan Wijnholds independently analysed the estimation of a single TID and found that to be well-conditioned.

- Not surprising: many, many fewer degrees of freedom in a sinusoid than the turbulent spectrum.
PPC: Four steps
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• Apply using AW Projection or similar
Solution for phase screen

- Write down all the constraint equations i.e. pierce point locations
- Form normal equations
- Eigenvalue/SVD analysis to get propagation of errors
Insight from A matrix

\[ A_{J,i} = \sqrt{N_{\text{station}}} \frac{S_i}{\sigma_{vis}} Z_j(r_i, \phi_i) \]

- 5 sources at 100MHz, 0.1 MHz
- Current proposed Low configuration
- Can see the redundancy of the spiral arms in Noll space
Estimates of degrees of freedom

• Very useful Noll formula for rms phase error due to unmodeled part of Kolmogorov spectrum

\[ \sigma_f^2 \sim 0.2944 J^{-\frac{\sqrt{3}}{2}} \left( \frac{B}{r_0} \right)^{\frac{5}{3}} \text{ rad}^2 \]

• In terms of highest singular value \( J \) fitted

• Gives dynamic range estimate

• Can use this to derive minimum \( J \) required to image to 50dB in 6 months (time on sky)

• 15,000 Zernikes

• Interpret this as 15,000 degrees of freedom!
“Zernike transform”
Response to comments

• Released response as SDP memo 27
• Use Bregman’s source counts in place of Condon’s.
• Corrected errors in noise calculation using tabulations in BD V2
• Updated diffractive scale from 14km to 7km (per recently published LOFAR work)
• Introduced superior ionosphere estimation approach
Direct Fitting to Visibilities

• Use brightest sources to stabilise the ionosphere to better than 1 radian

• Take linear approximation

\[ V(x_i, x_j) = 2\pi j \sum_S S e^{j\left(\frac{2\pi}{\lambda}(x_i-x_j) + \frac{\pi}{\lambda h}(x_i^2-x_j^2)\right)} \lambda^2 \left( T(x_i + h\sigma) - T(x_j + h\sigma) \right) \]

• Solve for TEC screen

• Apply using AWProjection

• Allows more sources to be used
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<th>Config</th>
<th>Bandwidth MHz</th>
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Other effects not yet included!!!!

- The relationship between phase and TEC
- Incomplete knowledge of the sky and antenna performance [RD7]
- Non-linear coupling and bias in the solution for phases of pierce points
- Non-linear coupling and bias in the peeling of bright sources
- Off-zenith primary beam effects
- The effect of sources outside the primary beam
- Other non-ideal behavior of the telescope (see e.g. [RD6])
- The strong variability seen in ionospheric behavior (see e.g. [RD1])
- Vertical structure in the ionosphere (i.e. multiple layers [RD9] and tubes [RD12])
- Algorithms to estimate and apply the phase screens, such as clustered calibration [RD8][RD14]
- Noise arising from the intra-station calibration
- Fresnel effects
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• O(100 - 200) years elapsed time for EOR

• Weaknesses of my analysis?
  
  • (-) Model primary beam sidelobes and peeling
  
  • (-) Off-zenith effects
  
  • (?) Connection between phase error and dynamic range
  
  • (?) Better handling of Fresnel effects
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- Main source of calibration errors is the ionosphere through which the telescope images

- Can the ionosphere be calibrated for EOR imaging?: No
- Can we design array to optimise calibration?: Yes, but does not help much
- Are we using the best calibration algorithms?: Still improvements to be made!
Escaping from my conclusions?
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• I would like to encourage more work in this analytical vein.
Final words

• We need more work on the analysis of data processing rather than just work on algorithms themselves

• The calibration of LOW is intrinsically difficult to understand and analyse

• My work is on the right track but requires further extension either in the current framework or in something more sophisticated

• Lots of opportunities for this audience!