Phase Noise Metrology

from RF to Photonics

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Outline

- Phase noise and variance
- Noise in Devices
- Linear Time-Invariant Systems
- Noise in Oscillators
- Measurements

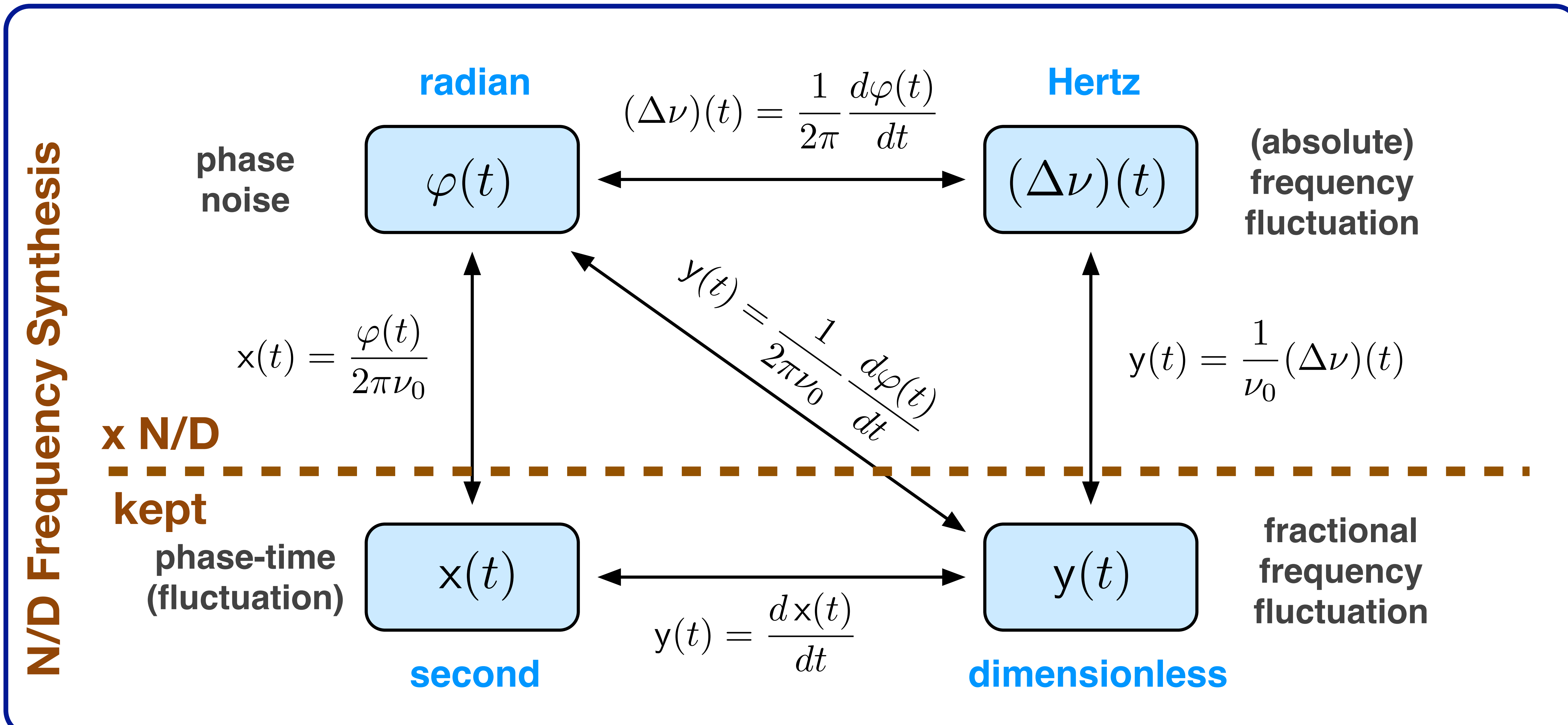
home page http://rubiola.org

Phase Noise and Wavelet (Allan) Variance

Physical Quantities

$$v(t) = V_0\,[1+\alpha(t)]\cos\,[2\pi\nu_0 t + \varphi(t)]$$

Allow φ(t) to exceed ±π and count the number of turns, so that φ(t) describes the clock fluctuation in full



$S_\varphi(f)$ and $\mathscr{L}(f)$

Phase noise PSD $S_\varphi(f)$

$$S_\varphi(f) = \mathcal{F}\left\{C_{\varphi\varphi}(\tau)\right\} \qquad \text{(Autocovariance)}$$

$$S_\varphi(f) = \mathbb{E}\left\{\Phi(f)\Phi^*(f)\right\} \qquad \text{(WK theorem)}$$

$$\boxed{S_\varphi(f) \approx \frac{1}{T}\left\langle \Phi(f)\Phi^*(f)\right\rangle_m \qquad \text{(measured)}}$$

Units

$S\varphi$ –> [rad²/Hz]

$10\ \mathrm{Log}_{10}(S\varphi)$ –> [dBad²/Hz]

The IEEE Std 1139-1999 defines $\mathscr{L}(f)$ as

$$\boxed{\mathscr{L}(f) = \frac{1}{2} S_\varphi(f)}$$

$(\mathscr{L})_{dB} = (S\varphi)_{dB} - 3\ \mathrm{dB}$

Units

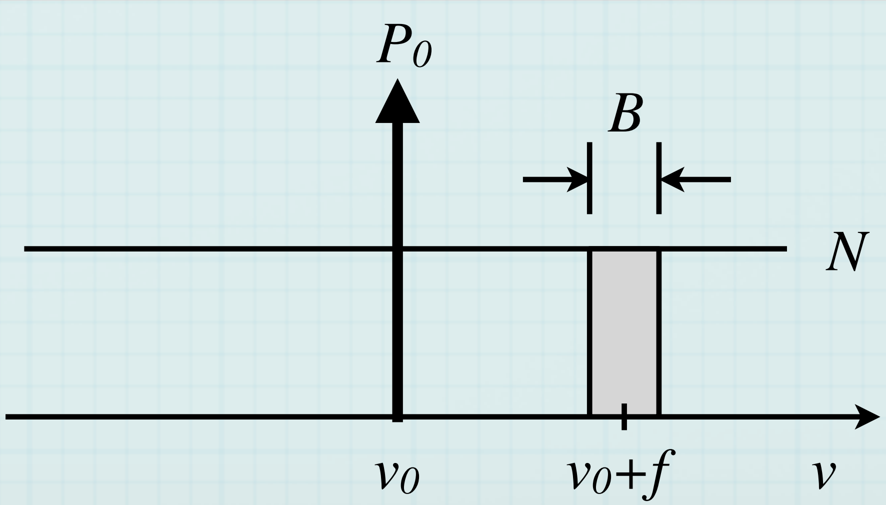
$10\ \mathrm{Log}_{10}(\mathscr{L})$ –> [dBc/Hz]

Unit of angle $\sqrt{2}$ rad ≈ 80°

The obsolete definition of $\mathscr{L}(f)$ is

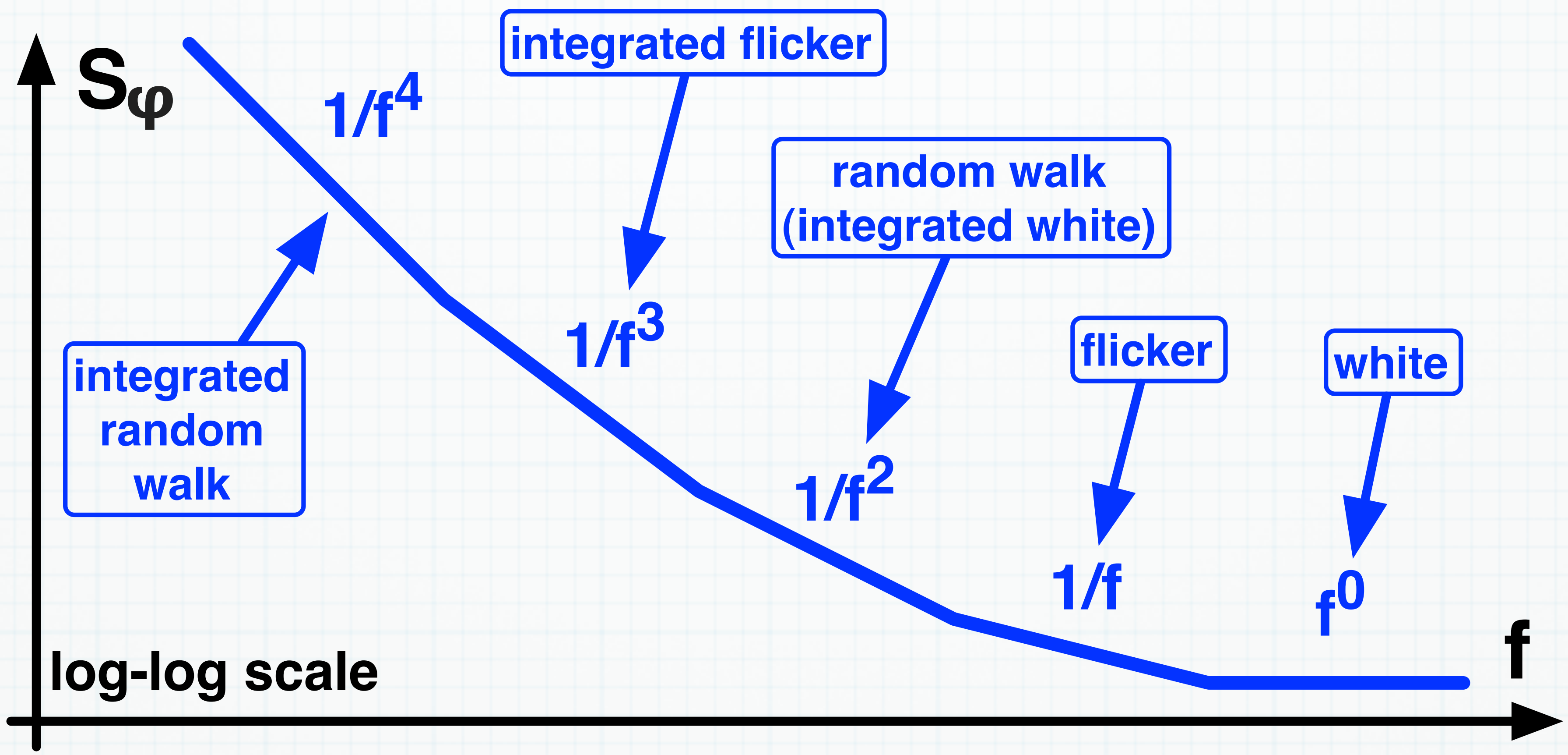
$$\mathscr{L}(f) = \frac{\text{SSB power in 1 Hz band}}{\text{carrier power}}$$

The problem with this definition is that it does not divide AM noise from PM noise, which yields to ambiguous results



Polynomial Law

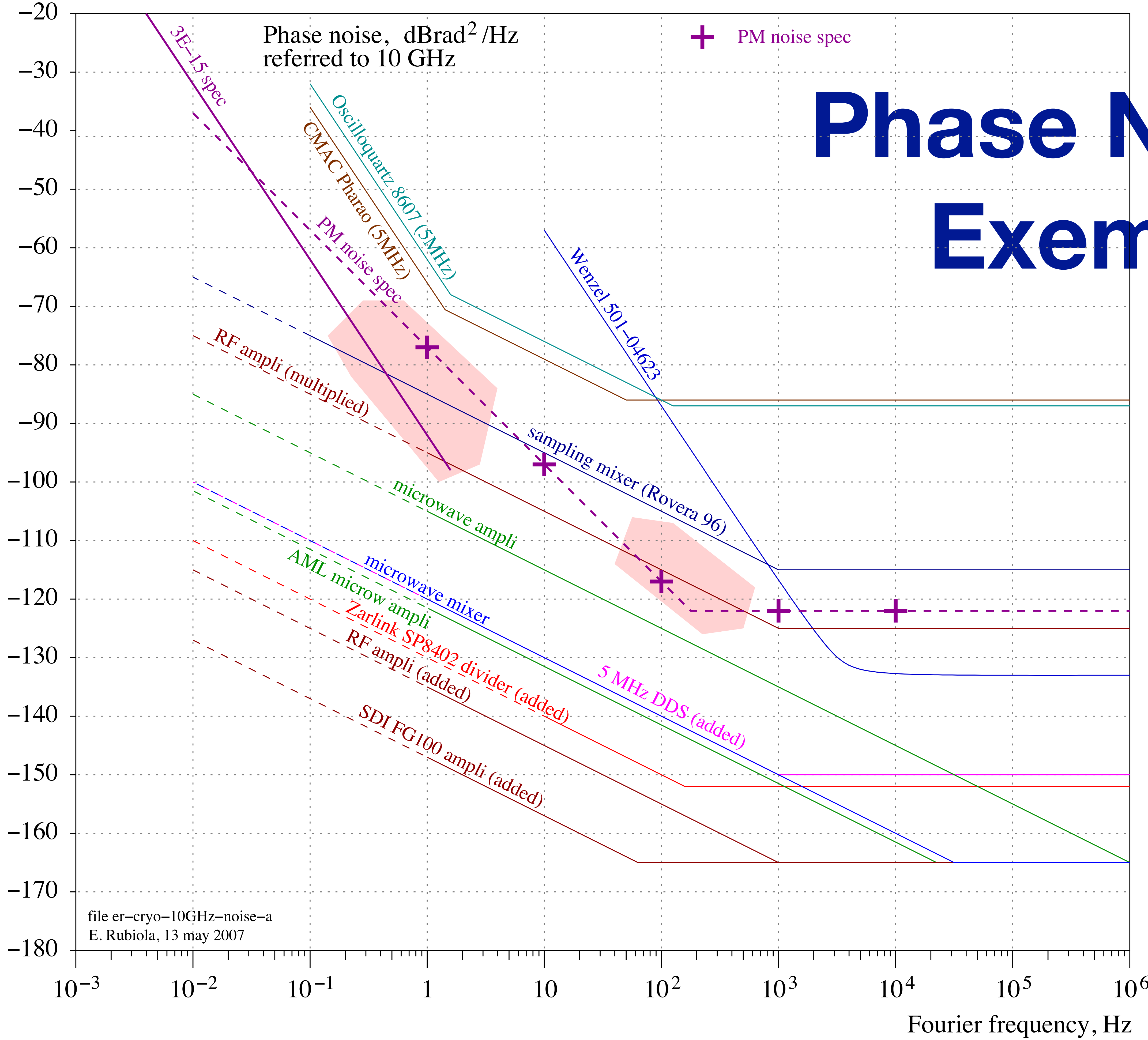
Power Spectral Density (PSD) $S_\varphi(f)$



Model which is useful to describe the close-in noise

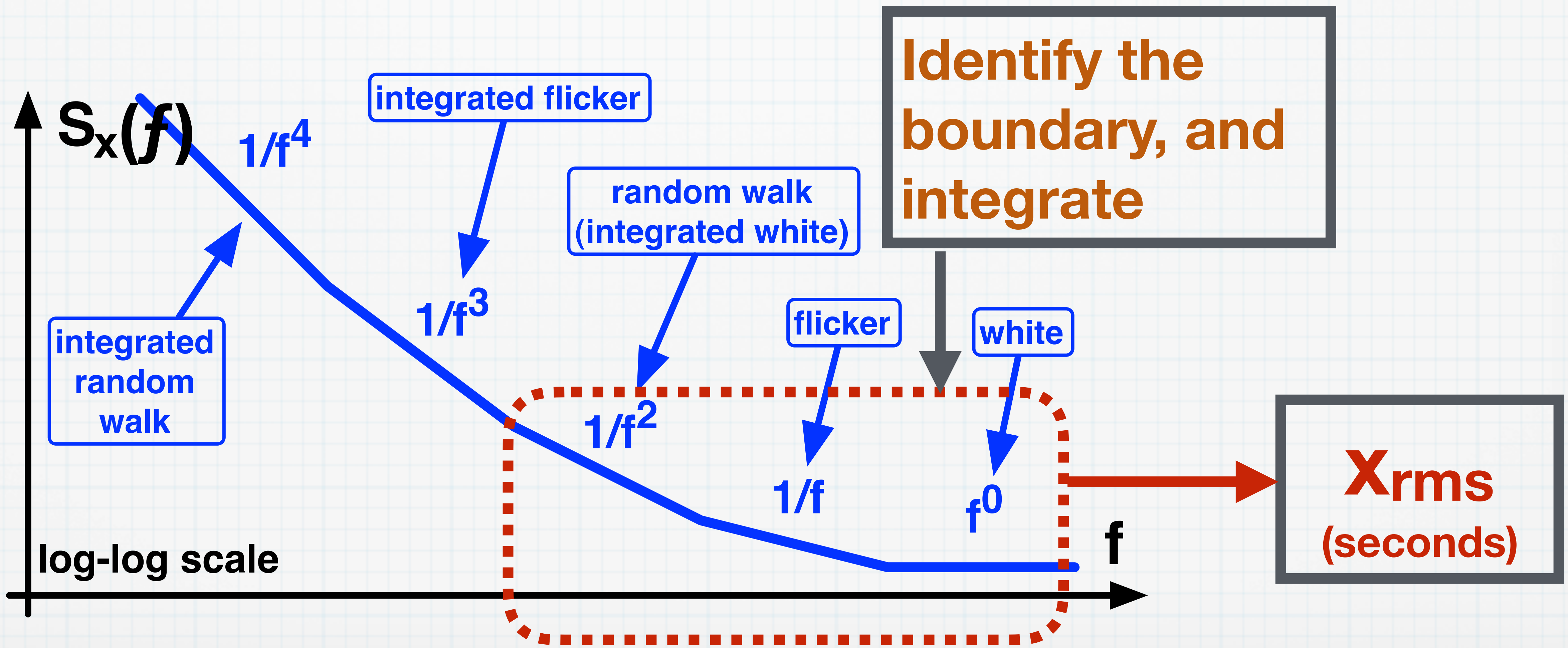
- Oscillator PM: $S_\varphi(f) = \ldots + b_{-4}/f^4 + b_{-3}/f^3 + b_{-2}/f^2 + b_{-1}/f + b_0$
- Oscillator FM: $S_y(f) = \ldots + h_{-2}/f^2 + h_{-1}/f + h_0 + h_1 f + h_2 f^2$
- Oscillator AM: $S_\alpha(f) = h_{-1}/f + h_0$ (chiefly, but not only)
- 2-port device PM: $S_\varphi(f) = b_{-1}/f + b_0$ (chiefly, but not only)
- 2-port device AM: $S_\alpha(f) = h_{-1}/f + h_0$ (chiefly, but not only)

Phase Noise Exemples



Jitter – Time Fluctuation

- Convert phase noise PSD into Sx(f)
  Phase-Time PSD
- Integrate over the suitable bandwidth
- Jitter bandwidth:
  - lower limit is set by the “size” of the system
  - upper limit is set by the circuit bandwidth



Flicker *Never* Diverges

$$P = \int_a^b S(f)\,df \qquad\qquad P = \int_a^b \frac{\mathsf{h}_{-1}}{f}\,df = \mathsf{h}_{-1}\,\ln\frac{b}{a}$$

$1/a = 4.3\text{x}10^{18}$ s (age of universe)

$1/b = 5.4\text{x}10^{-44}$ s (Planck time)

$\log_2(b/a) = 205.6$ (bits)

$\ln(b/a) \approx 142.5$ (21.5 dB)

However

Flicker introduces time correlation between samples (up to 1µs–1ms) and the averaging law $1/\sqrt{N}$ is gone

Allan Variance

definition $$\sigma_y^2(\tau) = \mathbb{E}\left\{\frac{1}{2}\left[\bar{y}_2 - \bar{y}_1\right]^2\right\}$$

expand as $$\sigma_y^2(\tau) = \mathbb{E}\left\{\frac{1}{2}\left[\frac{1}{\tau}\int_\tau^{2\tau} y(t)\,dt - \frac{1}{\tau}\int_0^{\tau} y(t)\,dt\right]^2\right\}$$

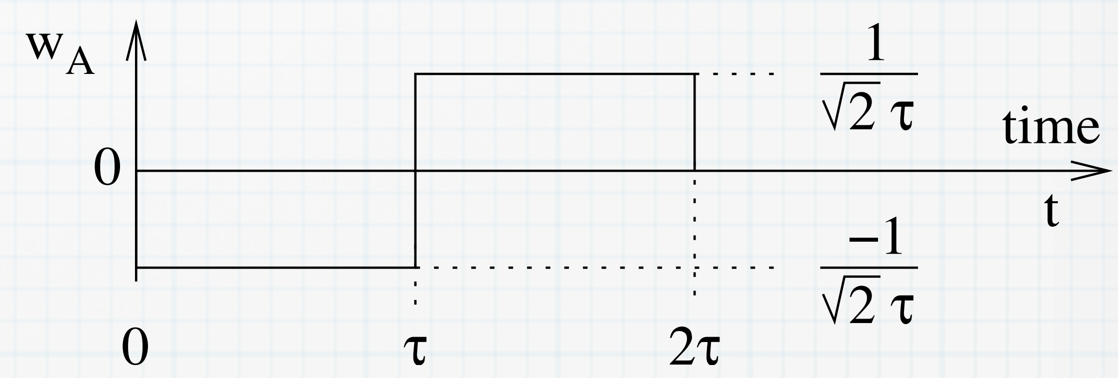
recall

$$\bar{y} = \frac{1}{\tau}\int_{t_0}^{t_0+\tau} y(t)\,dt$$

wavelet-like variance

$$\sigma_y^2(\tau) = \mathbb{E}\left\{\left[\int_{-\infty}^{+\infty} y(t)\,w_A(t)\,dt\right]^2\right\}$$

$$w_A = \begin{cases} -\frac{1}{\sqrt{2}\tau} & 0 < t < \tau \\ \frac{1}{\sqrt{2}\tau} & \tau < t < 2\tau \\ 0 & \text{elsewhere} \end{cases}$$



energy $$E\{w_A\} = \int_{-\infty}^{\infty} w_A^2(t)\,dt = \frac{1}{\tau}$$

the Allan variance differs from a wavelet variance in the normalization on power, instead of on energy

Phase Noise to AVAR Conversion

$$v(t) = V_p\,[1+\alpha(t)]\cos\left[2\pi\nu_0 t + \varphi(t)\right]$$

Phase noise

$$S_\varphi(f) = \mathbb{E}\left\{\Phi(f)\Phi^*(f)\right\} \quad \text{(expectation)}$$

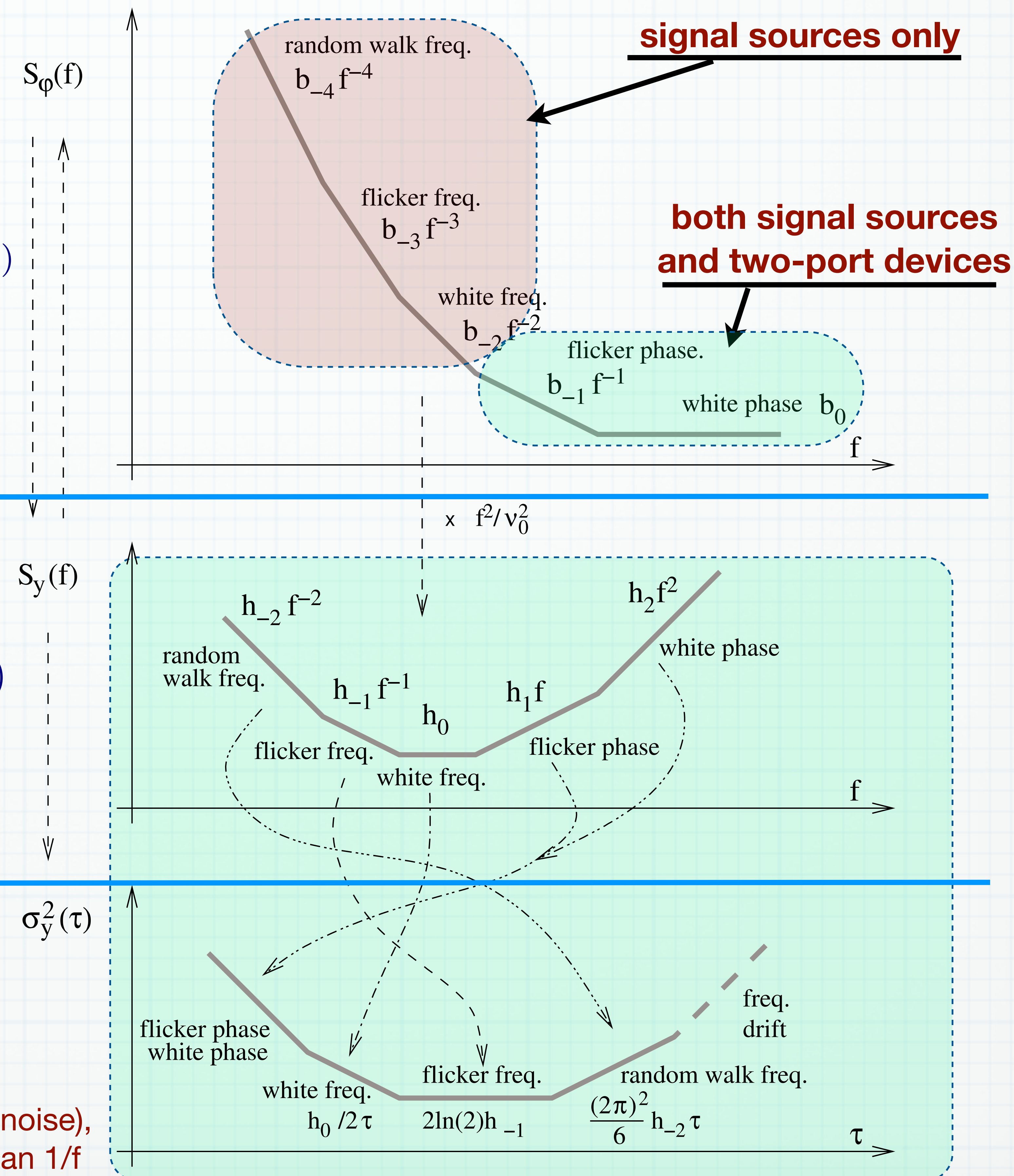
$$S_\varphi(f) \approx \left\langle \Phi(f)\Phi^*(f)\right\rangle_m \quad \text{(average)}$$

$$\mathsf{y}(t) = \frac{\dot\varphi(t)}{2\pi\nu_0} \quad\Rightarrow\quad S_\mathsf{y}(f) = \frac{f^2}{\nu_0^2} S_\varphi(f)$$

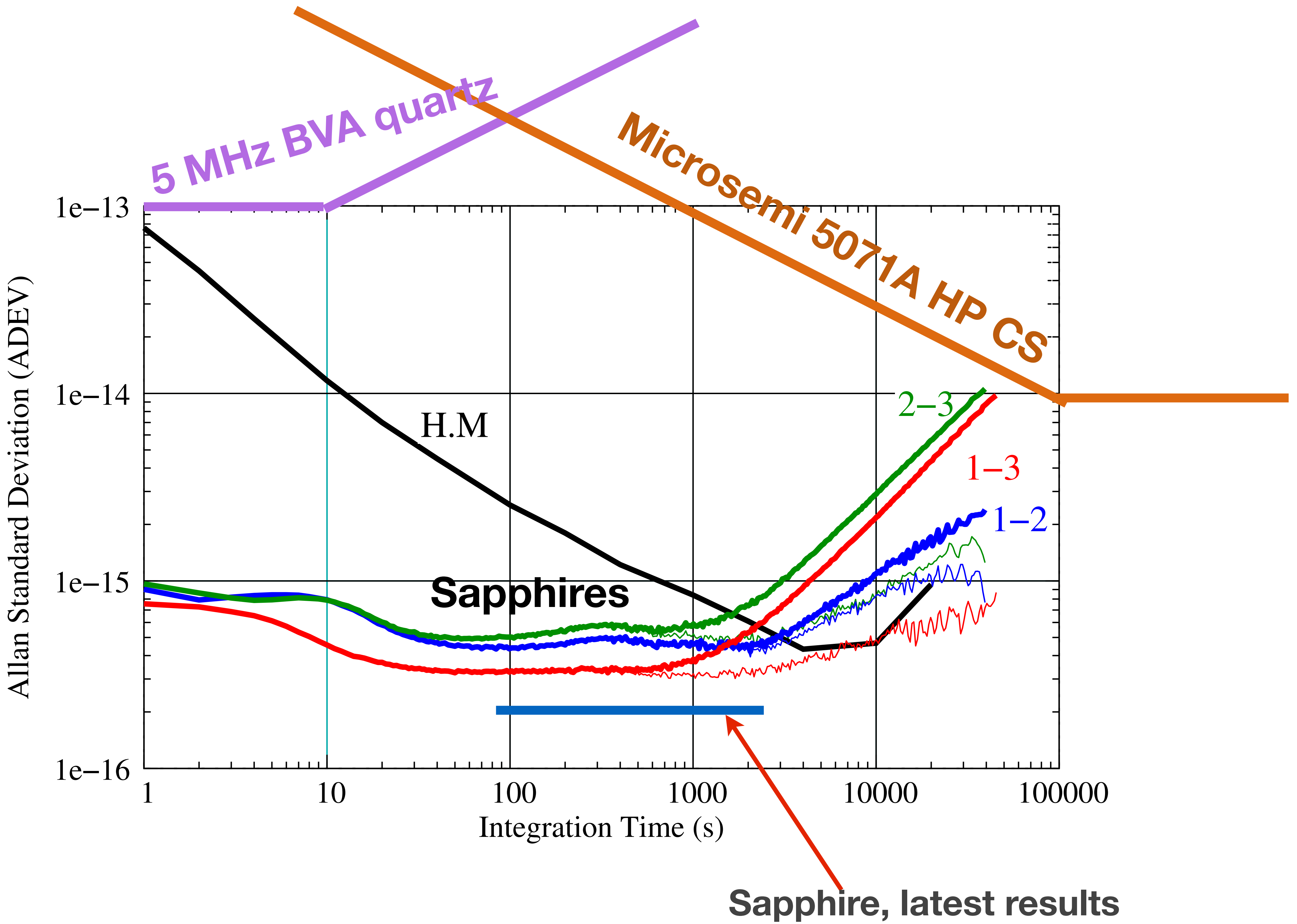
Allan variance
(two-sample wavelet-like variance)

$$\sigma_\mathsf{y}^2(\tau) = \mathbb{E}\left\{\frac{1}{2}\left[\overline{\mathsf{y}}_2 - \overline{\mathsf{y}}_1\right]^2\right\}$$

approaches a half-octave bandpass filter (for white noise),
hence it converges even with processes steeper than 1/f



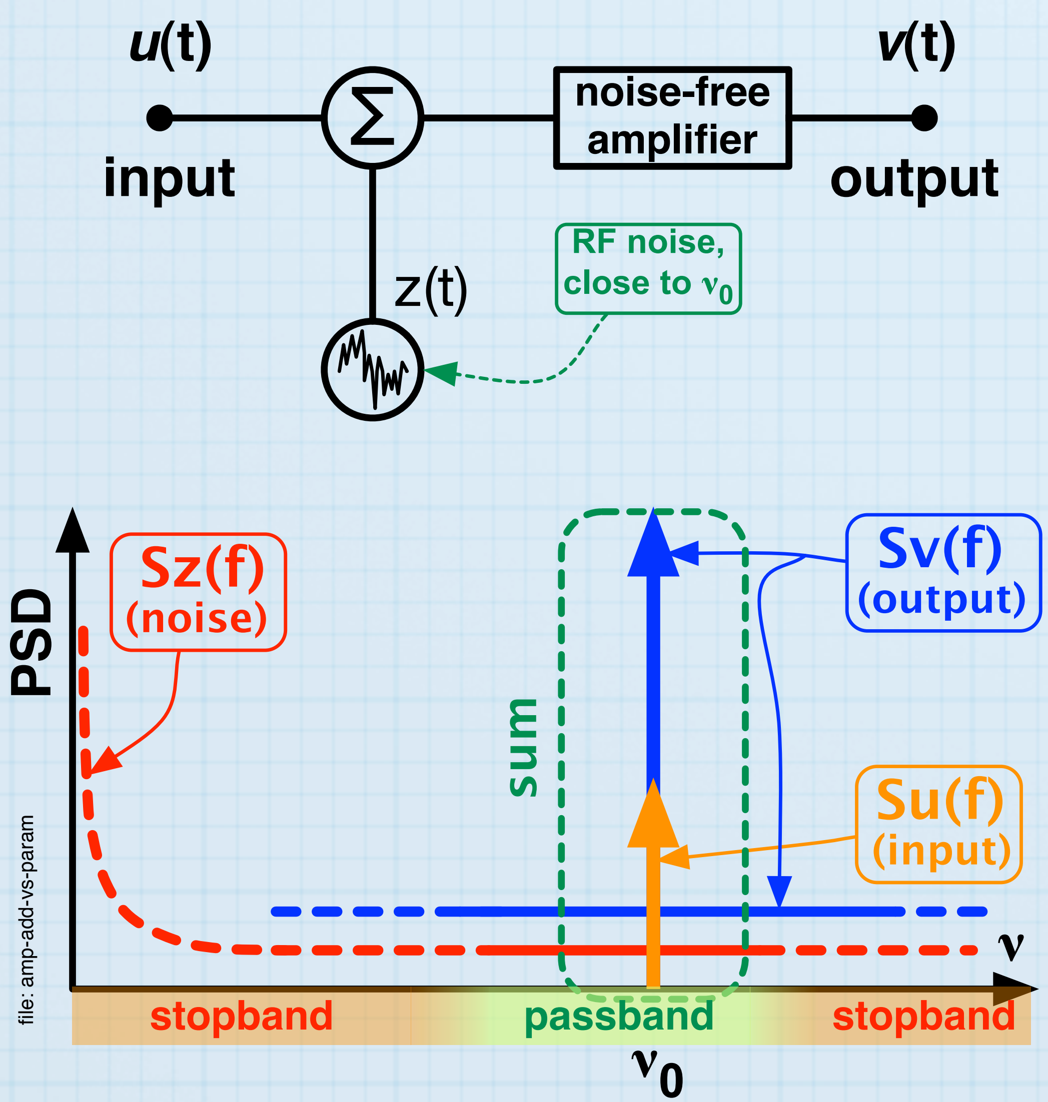
Allan Deviation – Examples



Phase Noise in Devices

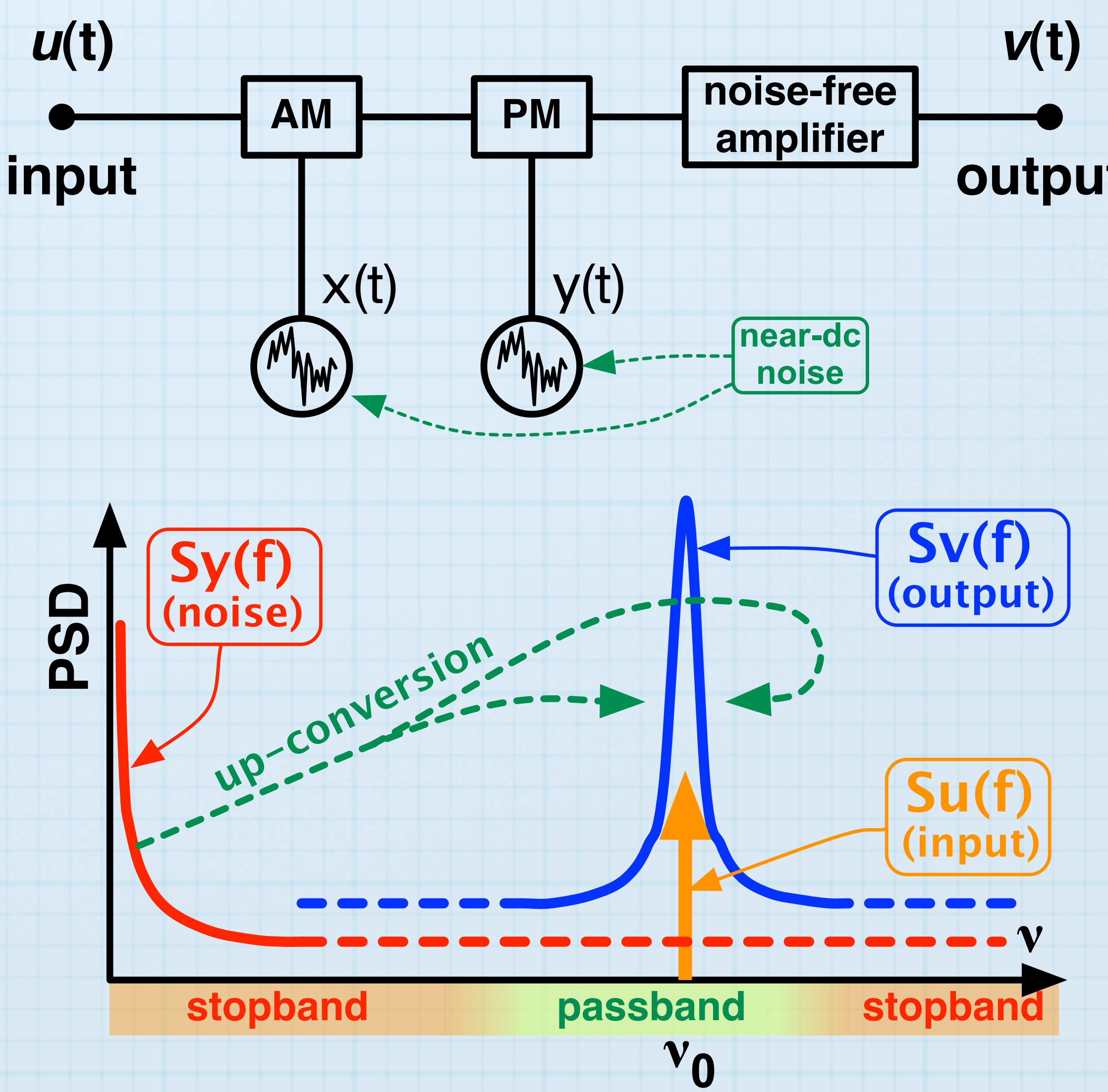
Additive VS Parametric Noise

additive noise



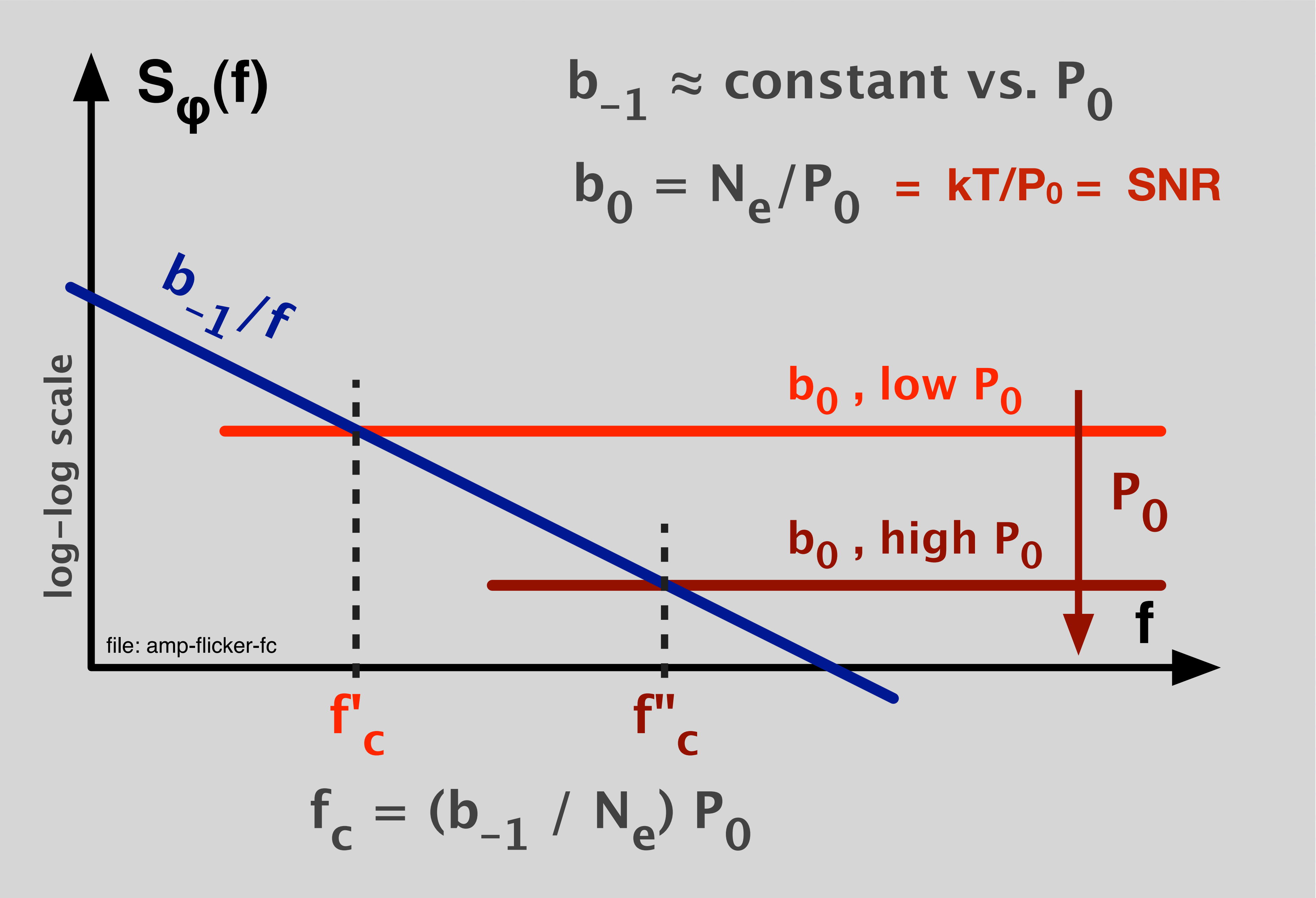
the noise sidebands are independent of the carrier

parametric noise



the noise sidebands are proportional to the carrier

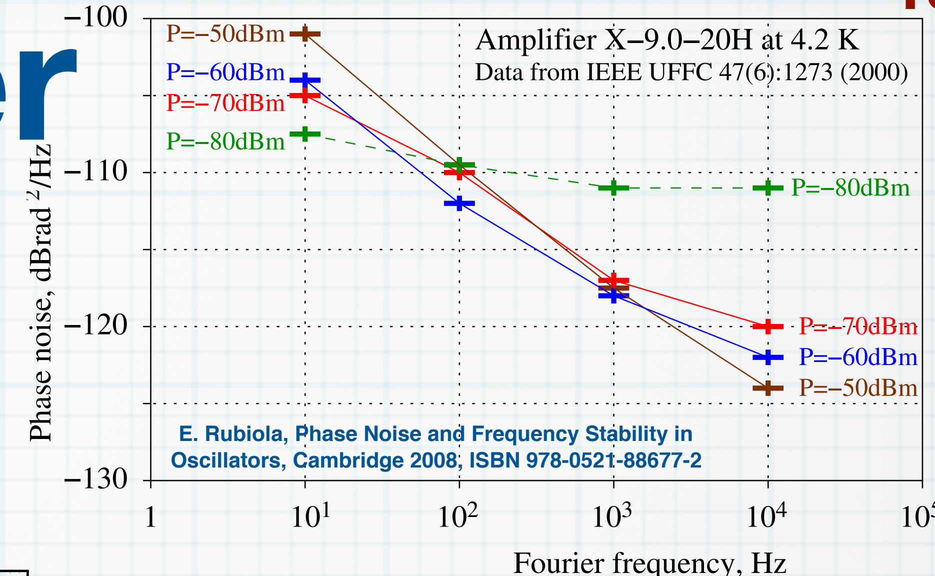
Amplifier White and Flicker Noise

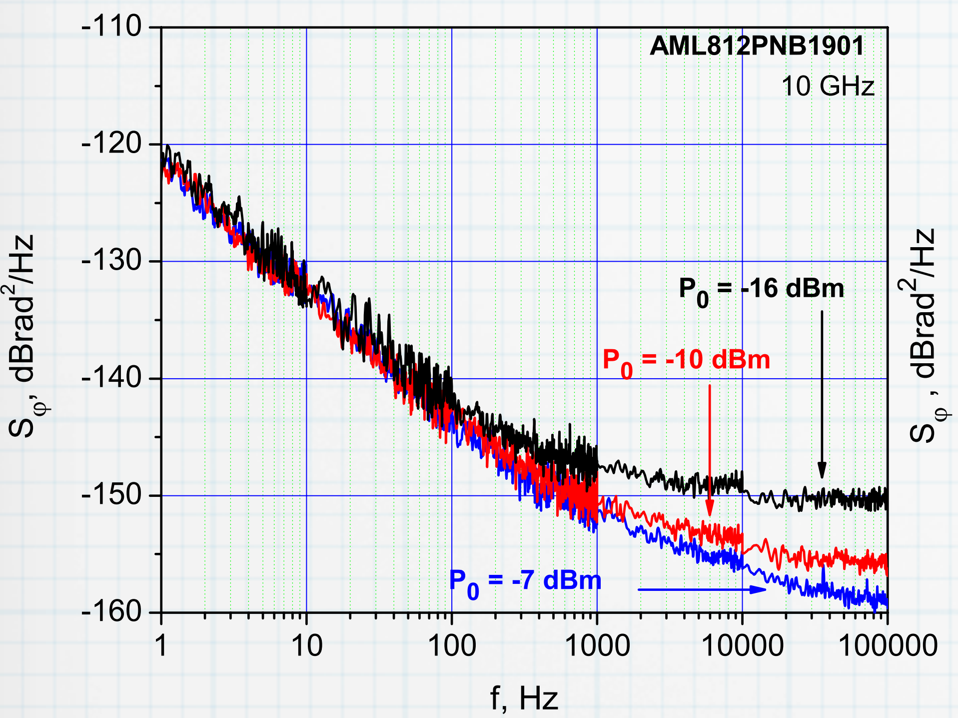


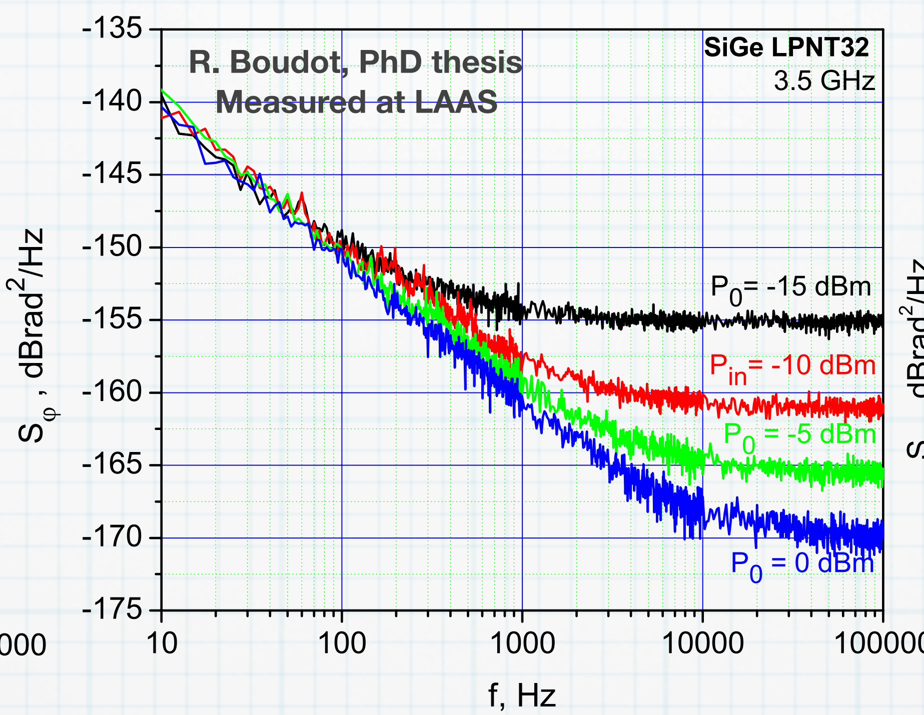
The corner frequency f_c, sometimes specified in data sheets is a misleading parameter because it depends on P_0

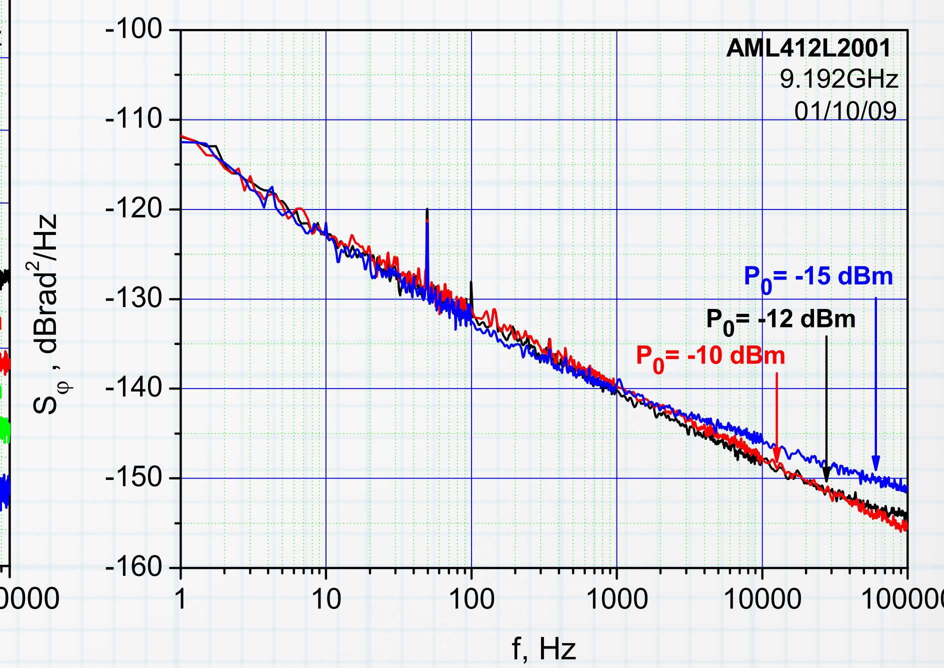
Phase noise vs. power

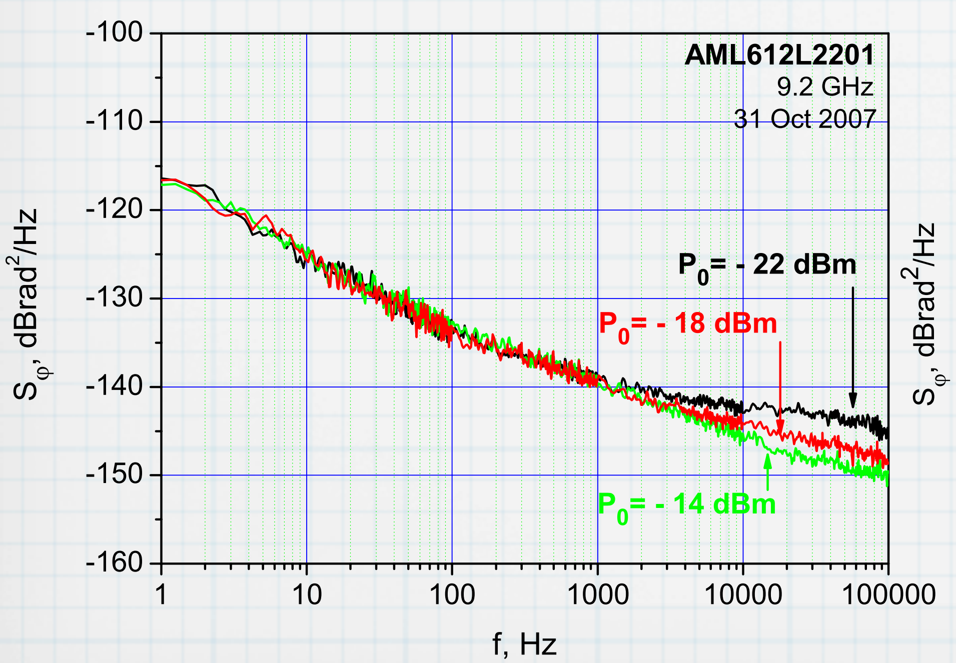
- The 1/f phase noise b–1 is about independent of power
- The white noise b0 scales as the inverse of the power
- The corner frequency is misleading because it depends on power

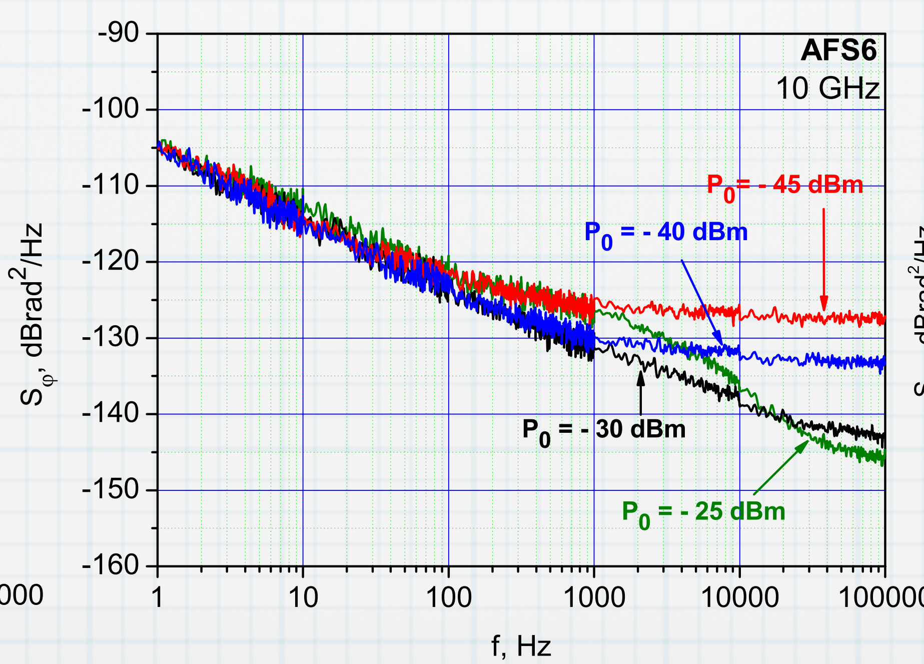


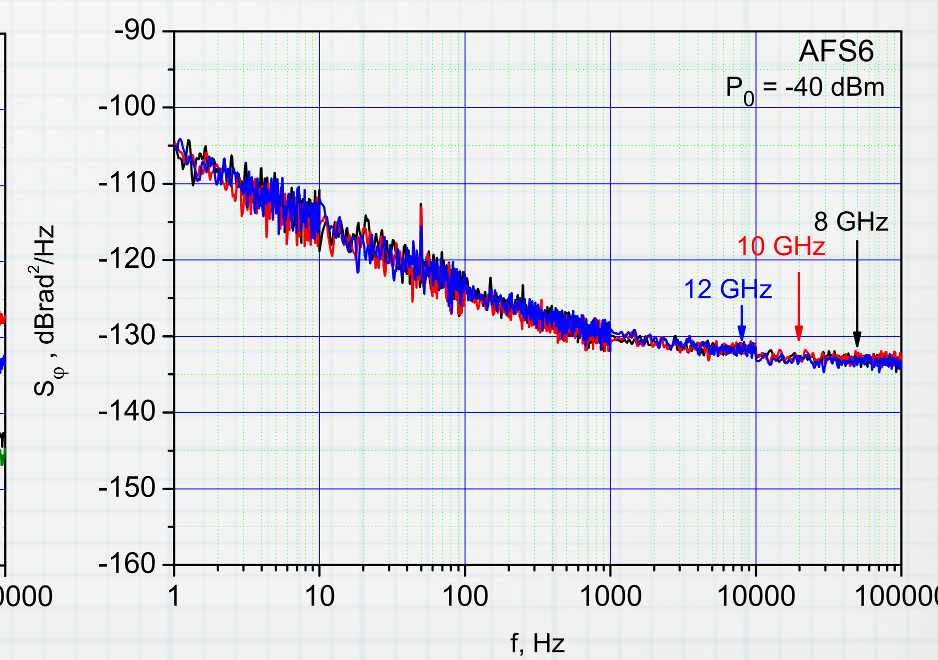




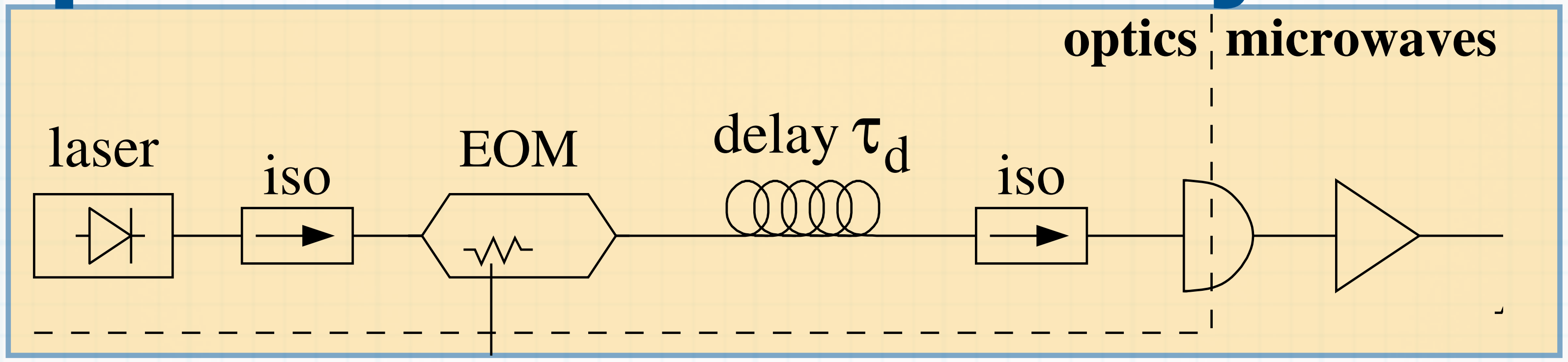








Opto-Electronic Delay Line



intensity modulation $P(t) = \overline{P}(1 + m\cos\omega_\mu t)$

photocurrent $i(t) = \dfrac{q\eta}{h\nu}\,\overline{P}(1 + m\cos\omega_\mu t)$

shot noise $N_s = 2\dfrac{q^2\eta}{h\nu}\,\overline{P}R_0$

microwave power $\overline{P}_\mu = \dfrac{1}{2}\,m^2 R_0\left(\dfrac{q\eta}{h\nu}\right)^2 P^2$

thermal noise $N_t = FkT_0$

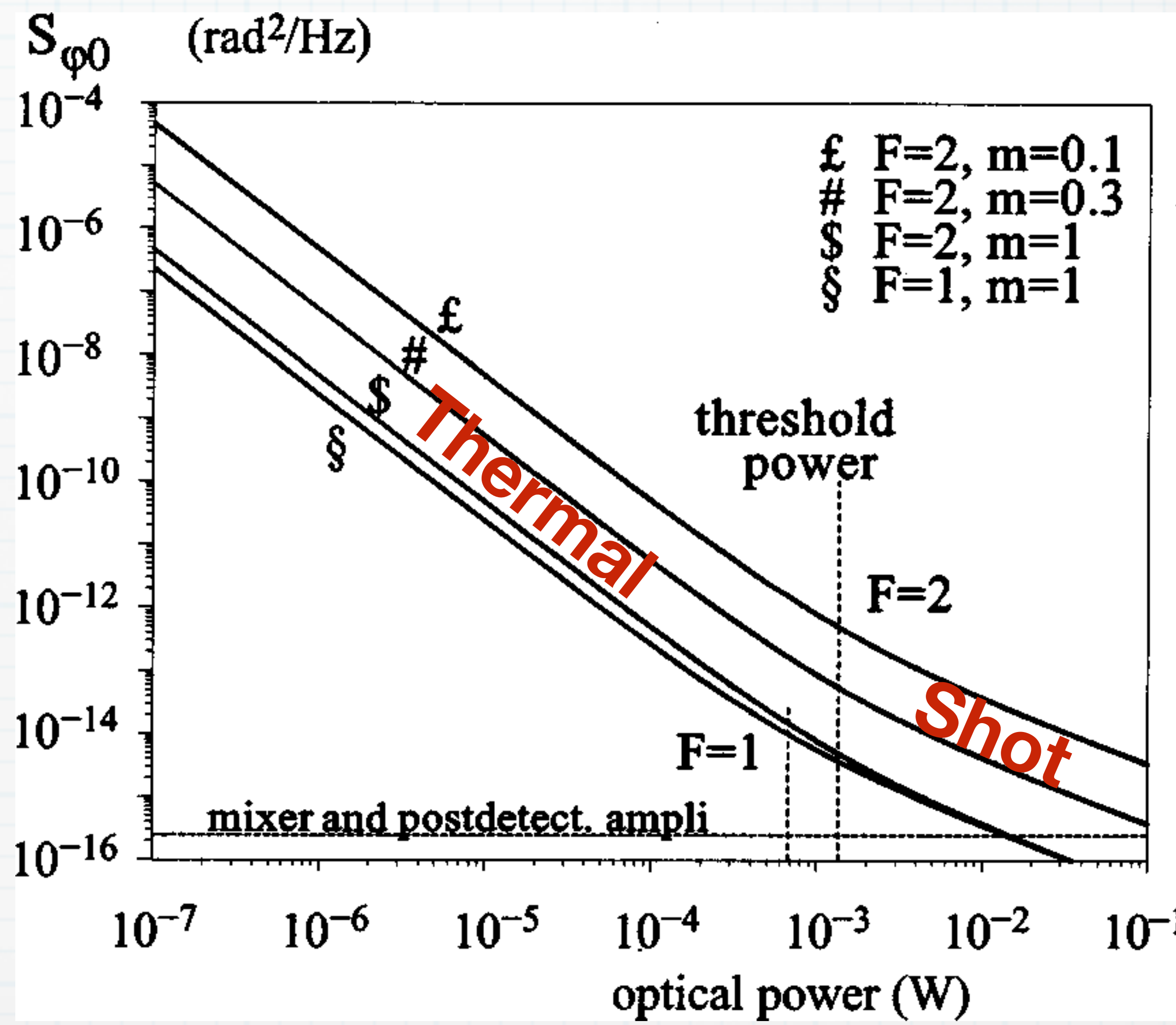
total white noise

$$S_{\varphi 0} = \frac{2}{m^2}\left[\underbrace{2\frac{h\nu_\lambda}{\eta}\frac{1}{\overline{P}}}_{\text{shot}} + \underbrace{\frac{FkT_0}{R_0}\left(\frac{h\nu_\lambda}{q\eta}\right)^2\left(\frac{1}{\overline{P}}\right)^2}_{\text{thermal}}\right]$$

flicker phase noise

- ampli: GaAs: $b_{-1} \approx -100$ to -110 dBrad²/Hz, SiGe: $b_{-1} \approx -120$ dBrad²/Hz
- photodetector $b_{-1} \approx -120$ dBrad²/Hz [Rubiola & al. MTT/JLT 54(2), feb. 2006
- (mixer $b_{-1} \approx -120$ dBrad²/Hz

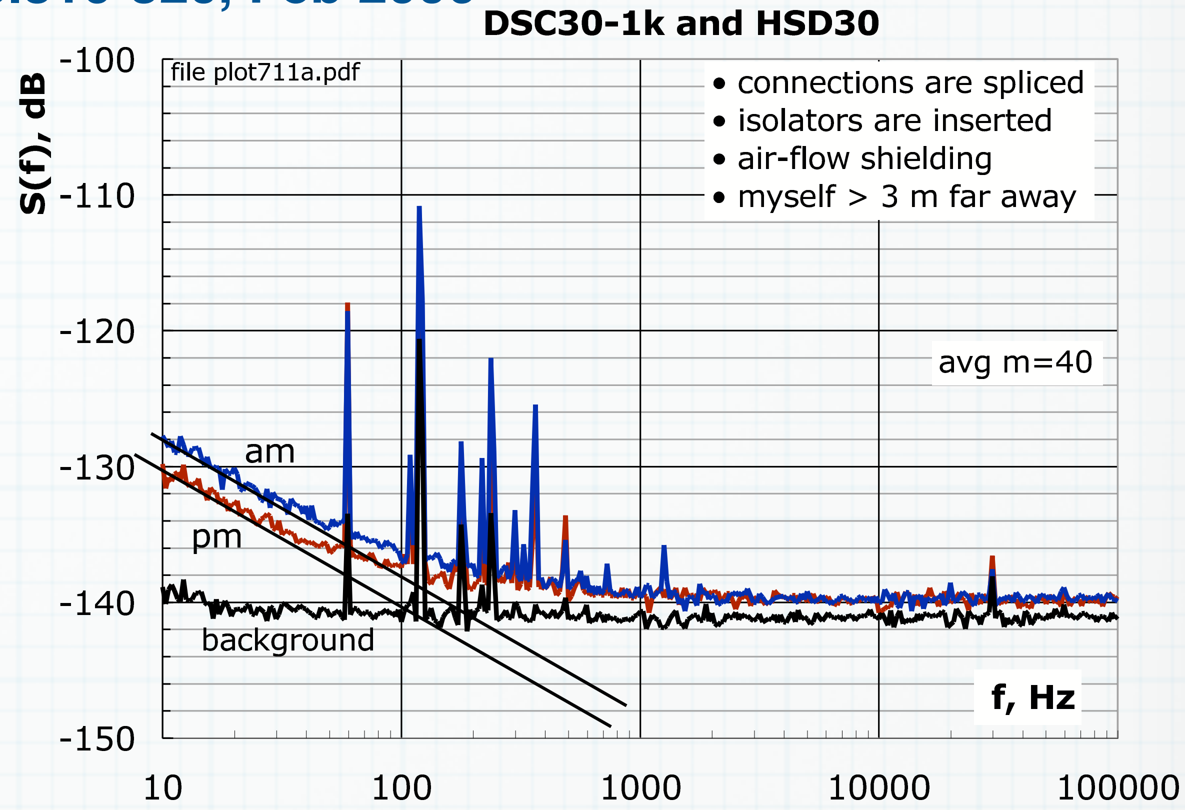
Optical-fiber phase noise?
Still an experimental parameter

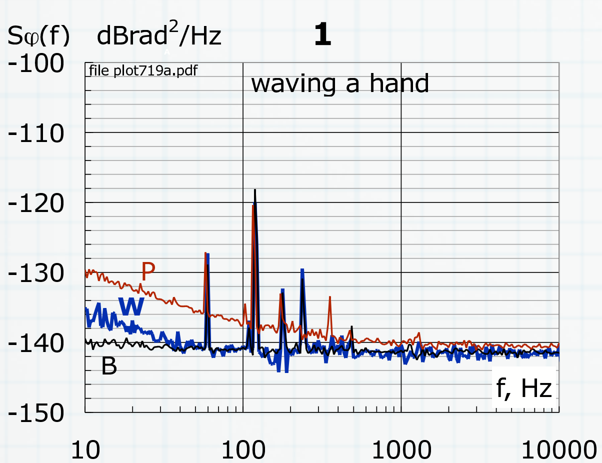


Photodetector 1/f noise

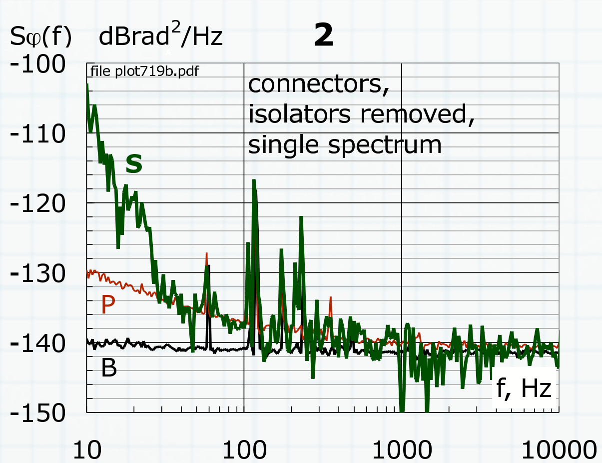
Rubiola, Salik, Yu, Maleki, MTT 54(2) p.816-820, Feb 2006

- the photodetectors we measured are similar in AM and PM 1/f noise
- the 1/f noise is about -120 dB[rad2]/Hz
- other effects are easily mistaken for the photodetector 1/f noise
- environment and packaging deserve attention in order to take the full benefit from the low noise of the junction

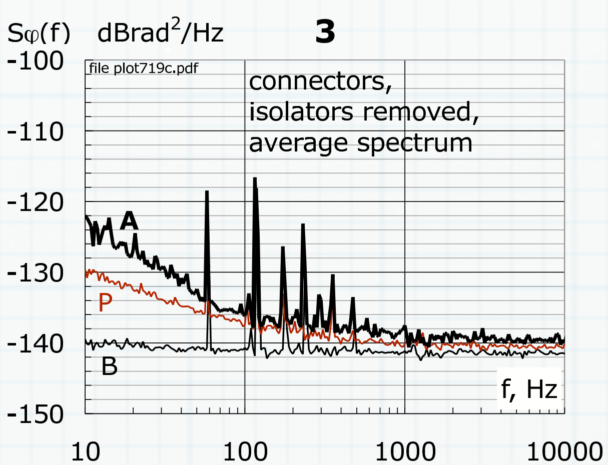




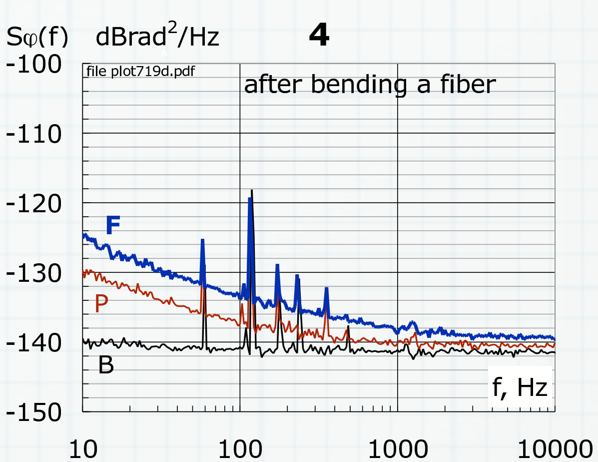
W: waving a hand 0.2 m/s, 3 m far from the system
B: background noise
P: photodiode noise



S: single spectrum, with optical connectors and no isolators
B: background noise
P: photodiode noise



A: average spectrum, with optical connectors and no isolators
B: background noise
P: photodiode noise



F: after bending a fiber, 1/f noise can increase unpredictably
B: background noise
P: photodiode noise

Physical phenomena in optical fibers

Birefringence. Common optical fibers are made of amorphous Ge-doped silica, for an ideal fiber is not expected to be birefringent. Nonetheless, actual fibers show birefringent behavior due to a variety of reasons, namely: core ellipticity, internal defects and forces, external forces (bending, twisting, tension, kinks), external electric and magnetic fields. The overall effect is that light propagates through the fiber core in a non-degenerate, orthogonal pair of axes at different speed. Polarization effects are strongly reduced in polarization maintaining (PM) fibers. In this case, the cladding structure stresses the core in order to increase the difference in refraction index between the two modes.

Polarization mode dispersion (PMD). This effect rises from the random birefringence of the optical fiber. The optical pulse can choose many different paths, for it broadens into a bell-like shape bounded by the propagation times determined by the highest and the lowest refraction index. Polarization vanishes exponentially along the light path. It is to be understood that PMD results from the vector sum over multiple forward paths, for it yields a well-shaped dispersion pattern.

Bragg scattering. In the presence of monocromatic light (usually X-rays), the periodic structure of a crystal turns the randomness of scattering into an interference pattern. This is a weak phenomenon at micron wavelengths because the inter-atom distance is of the order of 0.3--0.5 nm. Bragg scattering is not present in amorphous materials.

Brillouin scattering. In solids, the photon-atom collision involves the emission or the absorption of an acoustic phonon, hence the scattered photons have a wavelength slightly different from incoming photons. An exotic form of Brillouin scattering has been reported in optical fibers, due to a transverse mechanical resonance in the cladding, which stresses the core and originates a noise bump on the region of 200--400 MHz.

Raman scattering. This phenomenon is similar to Rayleigh scattering, but it involves the optical branch of phonons.

Rayleigh scattering. This is random scattering due to molecules in a disordered medium, by which light looses direction and polarization. In SM fibers at 1.55 μm it contributes 0.15 dB/km to the optical loss.

Double Rayleigh scattering. A small fraction of the light intensity is back-scattered, and in turn a $(\text{small})^2$ fraction is forward scattered. This is a stochastic to-and-fro path, which originates phase noise.

Kerr effect. This effect states that an electric field changes the refraction index. So, the electric field of light modulate the refraction index, which originates the 2nd-order nonlinearity.

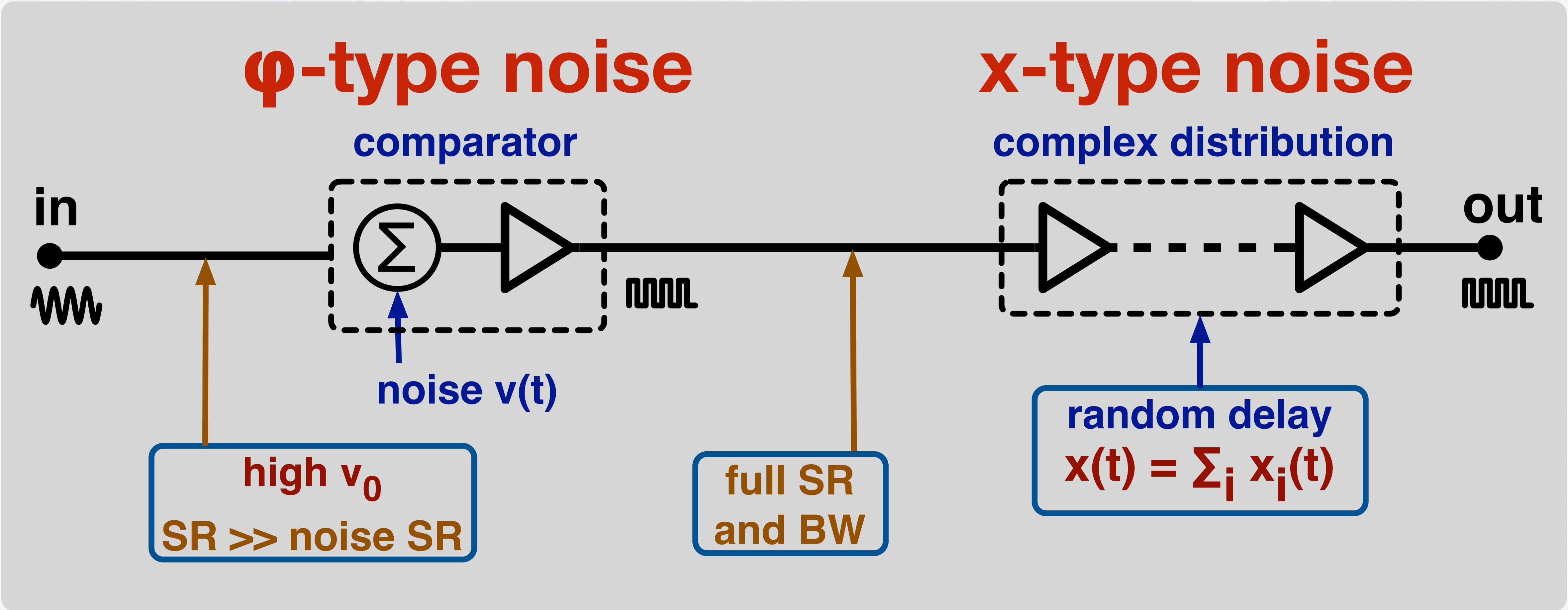
Discontinuities. Discontinuities cause the wave to be reflected and/or to change polarization. As the pulse can be split into a pulse train depending on wavelength, this effect can turn into noise.

Group delay dispersion (GVD). There exist dispersion-shifted fibers, that have a minimum GVD at 1550 nm. GVD compensators are also available.

PMD-Kerr compensation. In principle, it is possible that PMD and Kerr effect null one another. This requires to launch the appropriate power into each polarization mode, for two power controllers are needed. Of course, this is incompatible with PM fibers.

Which is the most important effect? In the community of optical communications, PMD is considered the most significant effect. Yet, this is related to the fact that excessive PMD increases the error rate and destroys the eye pattern of a channel. In the case of the photonic oscillator, the signal is a pure sinusoid, with no symbol randomness.

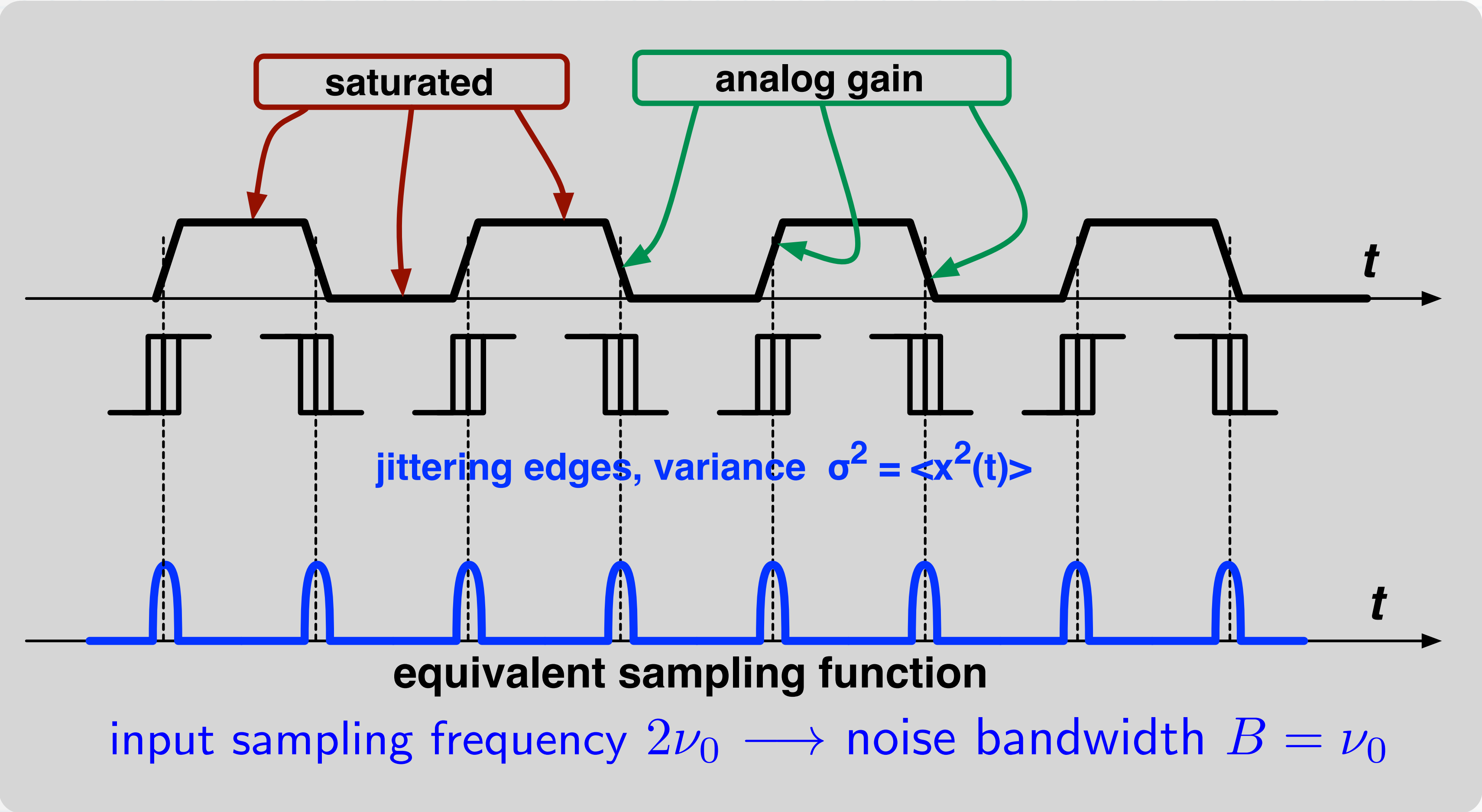
Basic Noise Mechanisms

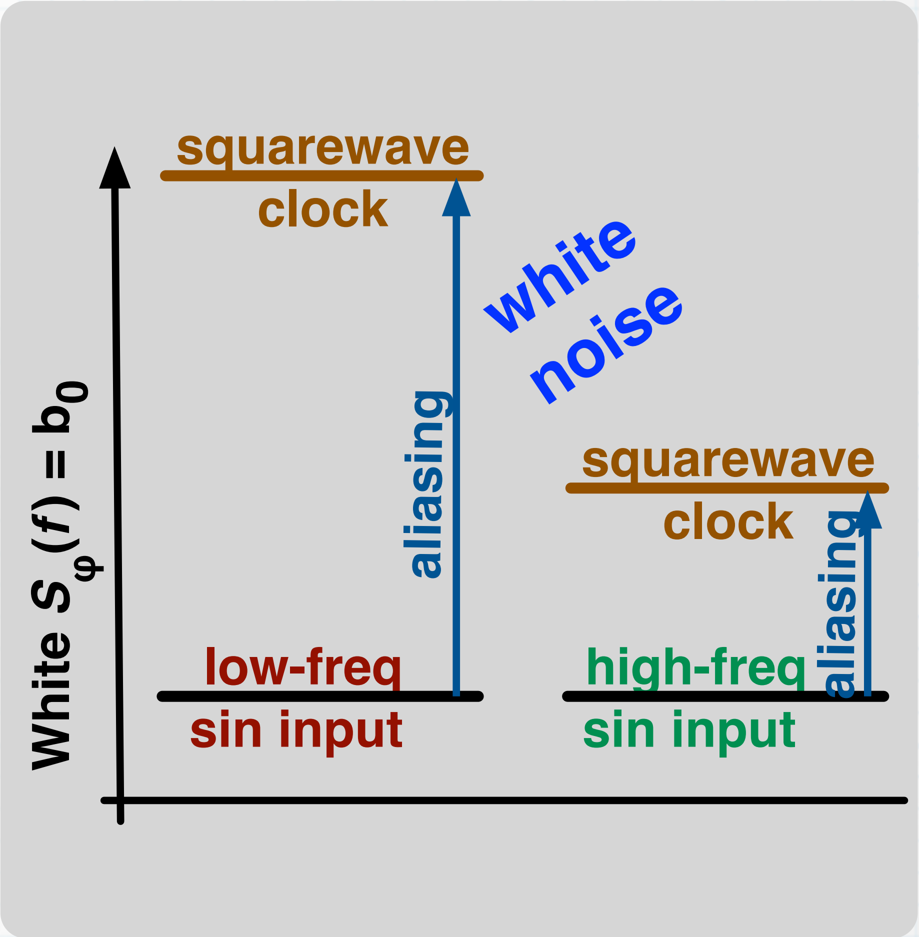


- The φ-type noise noise may show up or not, depending on input noise and SR
- At the comparator out, the edges attain full SR and bandwidth of the technology
- Complex distribution –> independent fluctuations add up

$$x(t) = \sum_i x_i(t) \quad \text{and} \quad \langle x^2(t)\rangle = \sum_i \langle x_i^2(t)\rangle$$

Aliasing Mechanism





Flicker and slow noise types

- Too low power at high frequency
- No aliasing

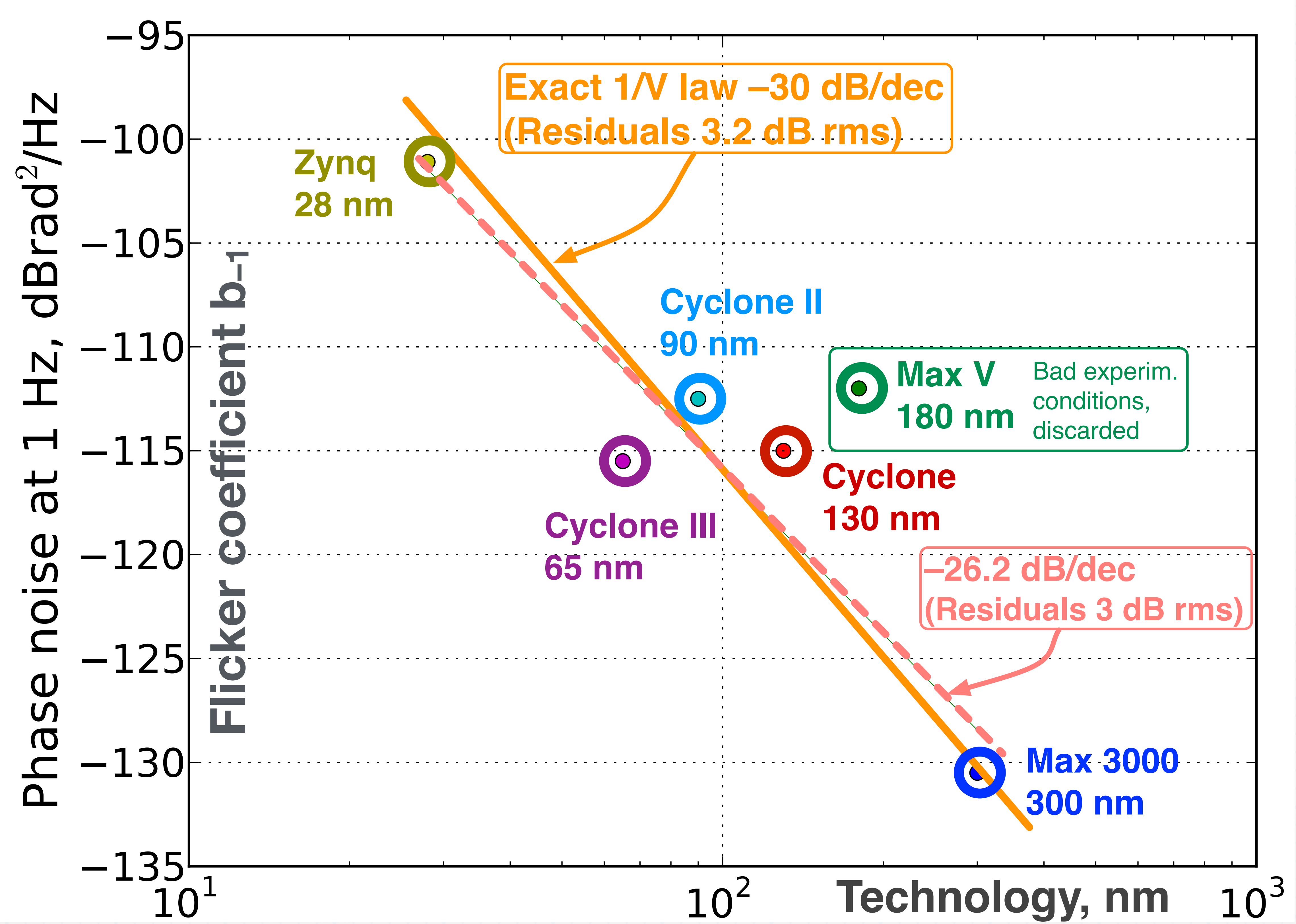
White noise

- The variance σ^2 is independent of frequency
- Parseval theorem applies

$$\sigma^2 = b_0 B = b_0 \nu_0$$

- Aliasing –> higher phase noise at lower carrier frequency

The Volume Law!

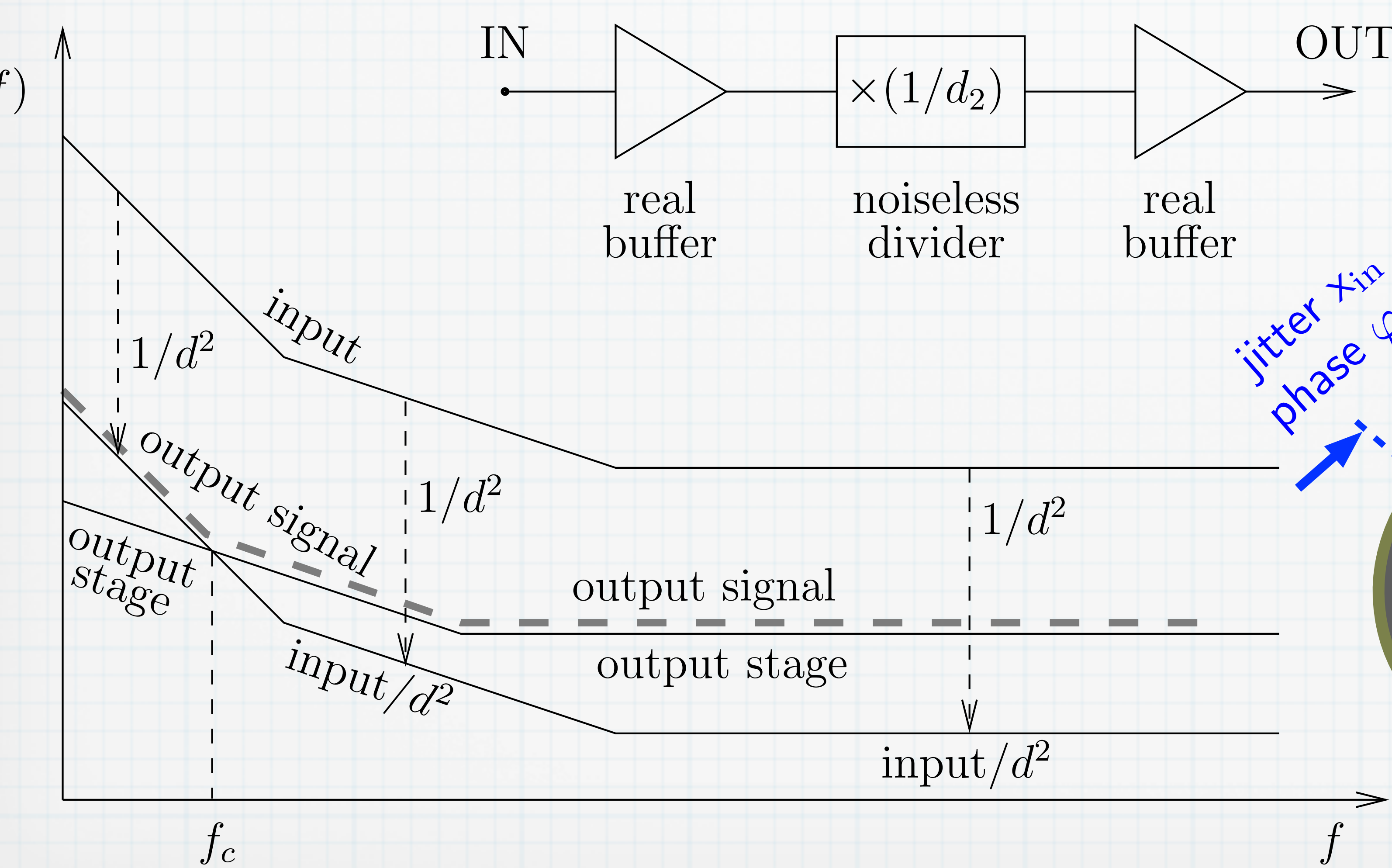


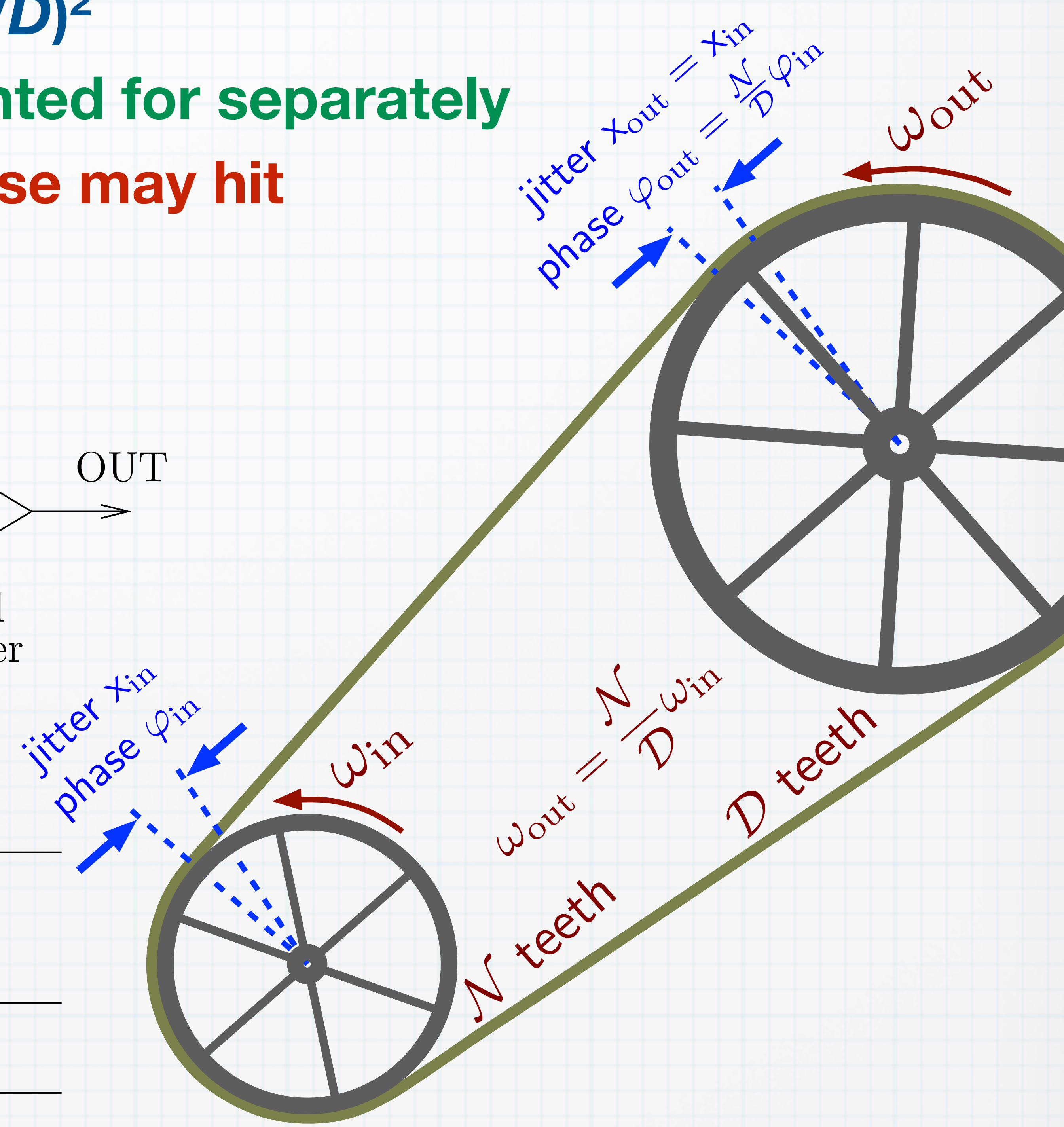
Details in file DevicesComparison.doc

Phase Noise in Synthesizers and Linear Time-Invariant Systems

The Noise-Free Synthesizer

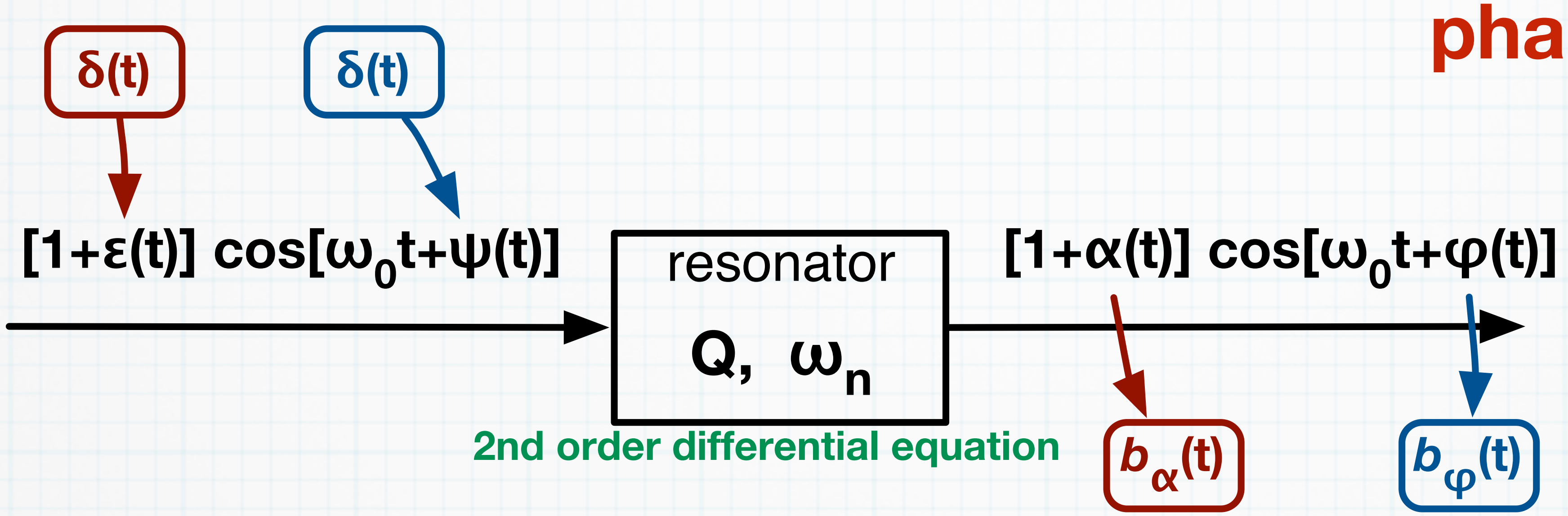
- The noise-free synthesizer propagates the jitter x (phase time)
  - So, it scales the phase φ as *N/D*,
  - and the phase spectrum S_φ as $(N/D)^2$
- Sampling (digital circuits) is accounted for separately
- In dividers (N/D<<1), the output noise may hit
  - the thermal floor
  - the noise of the output stage





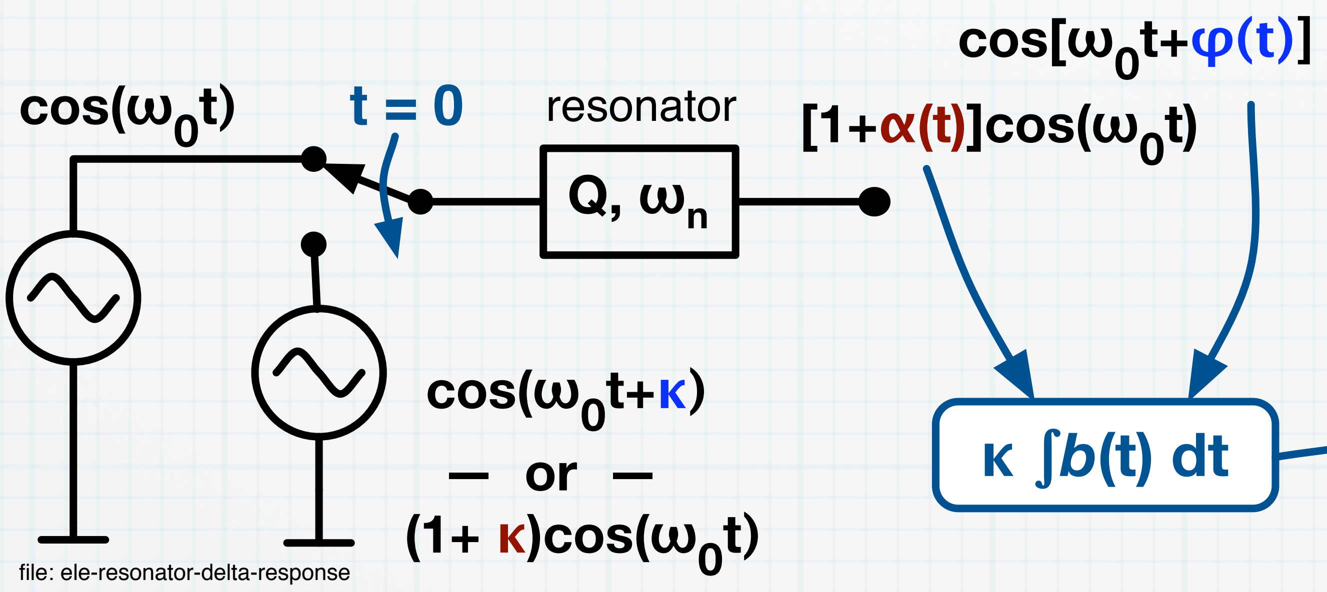
Resonator Impulse Response

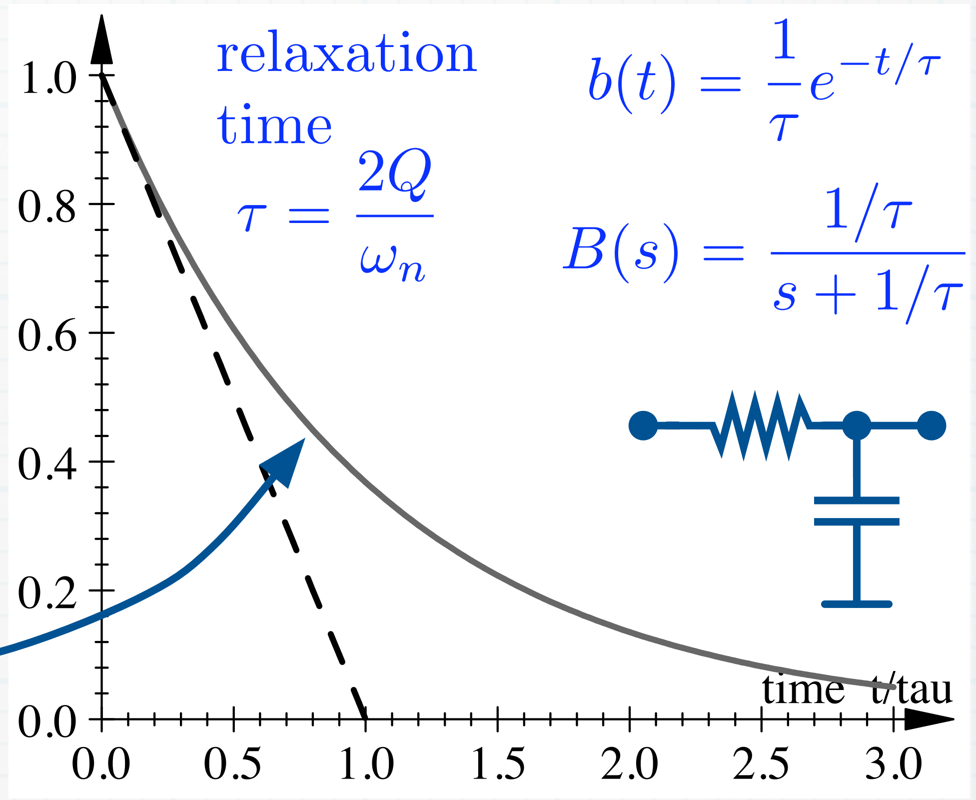
And a general method to solve phase noise problems



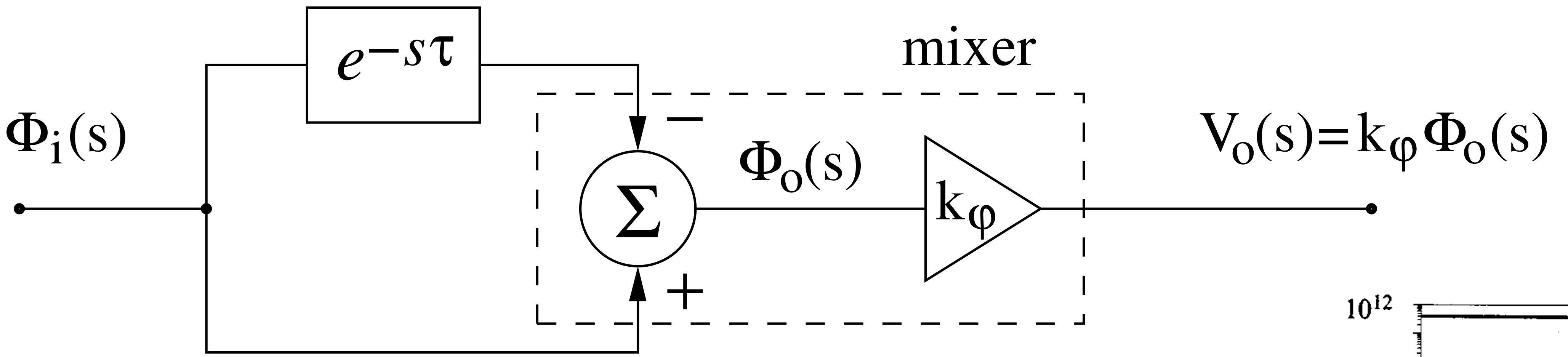
Can't figure out a δ(t) of phase or amplitude? Use Heaviside (step) u(t) and differentiate

set a small phase or amplitude step κ at t=0, and linearize for κ→0



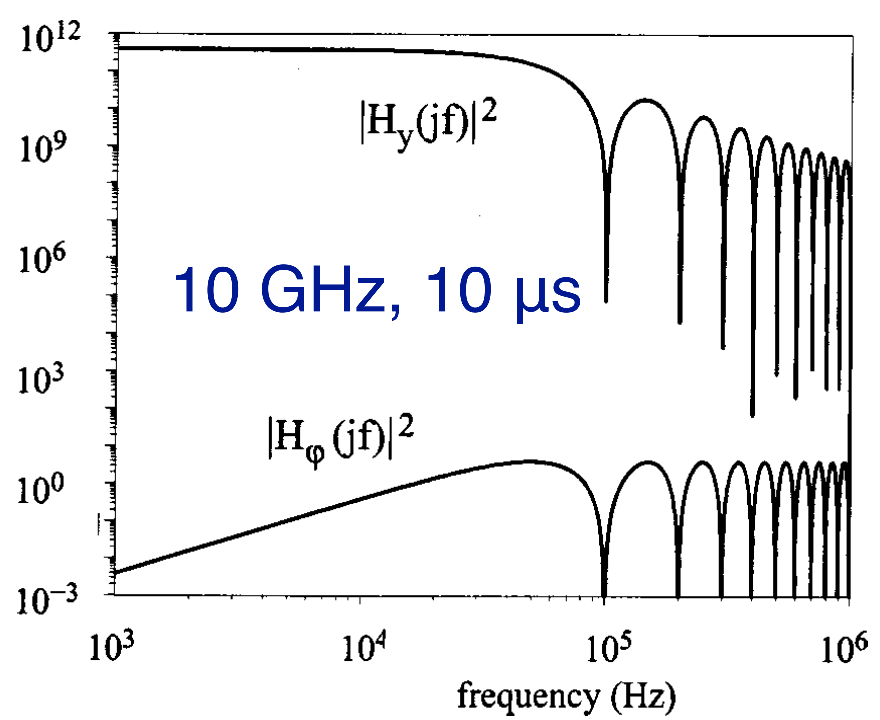


LTI Systems —> Transfer Function



$$\Phi_o(s) = (1-e^{-s\tau})\,\Phi_i(s)$$

$\approx s\tau\Phi$ for $s\tau \ll 1$

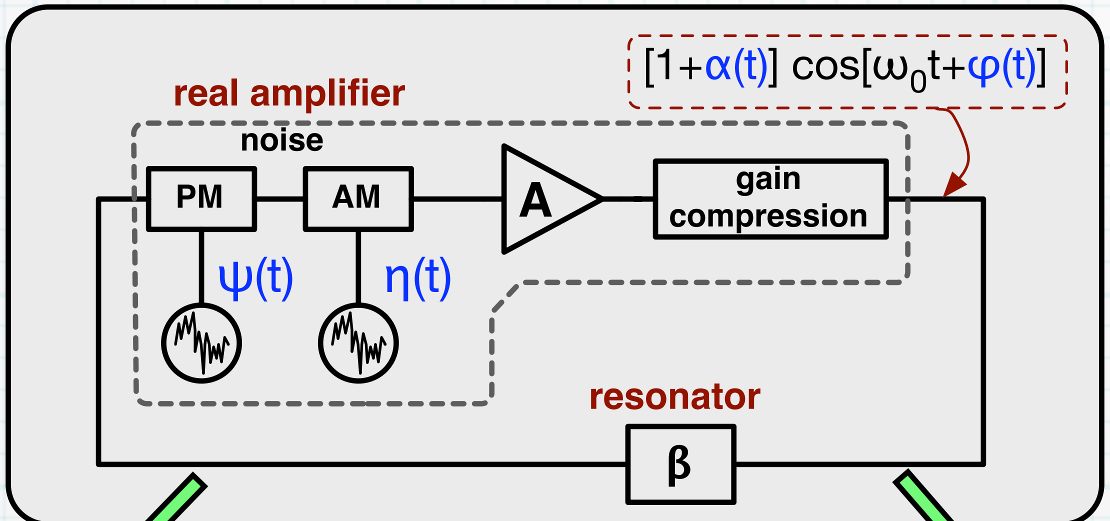


- **The effect of a delay line is shown**
- **All signals are the Laplace transform of the phase in the actual circuit**
- **This pattern is useful for the synchronization in the presence of a delay**

The Leeson Effect

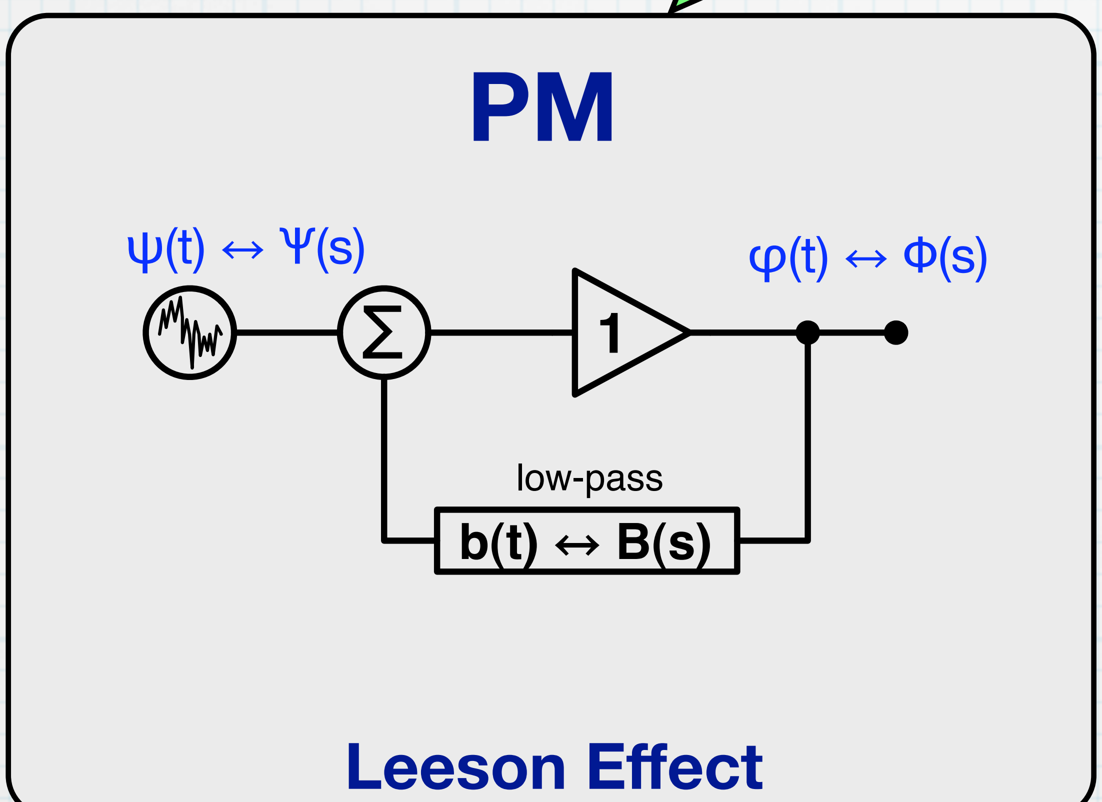
Phase Noise and Frequency Stability in Oscillators

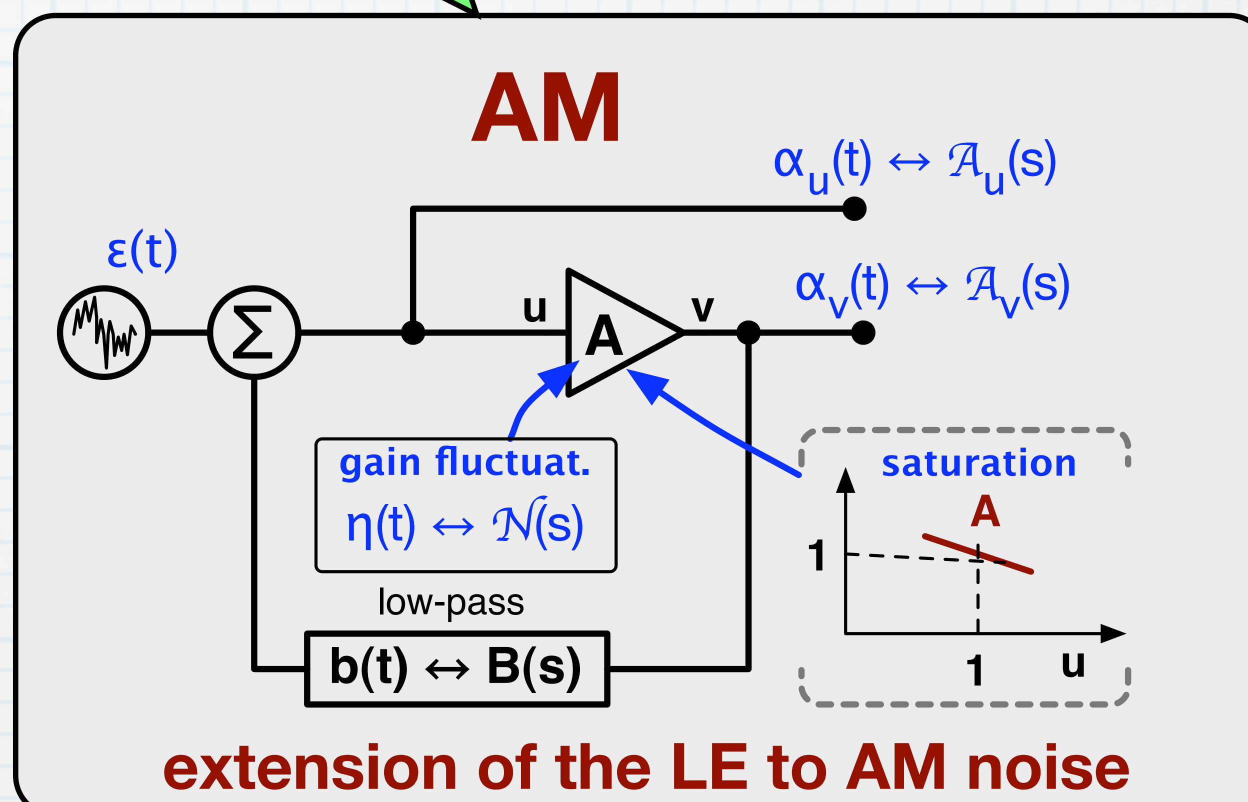
Low-Pass Representation of AM-PM Noise



RF, μwaves or optics

low-pass equivalent

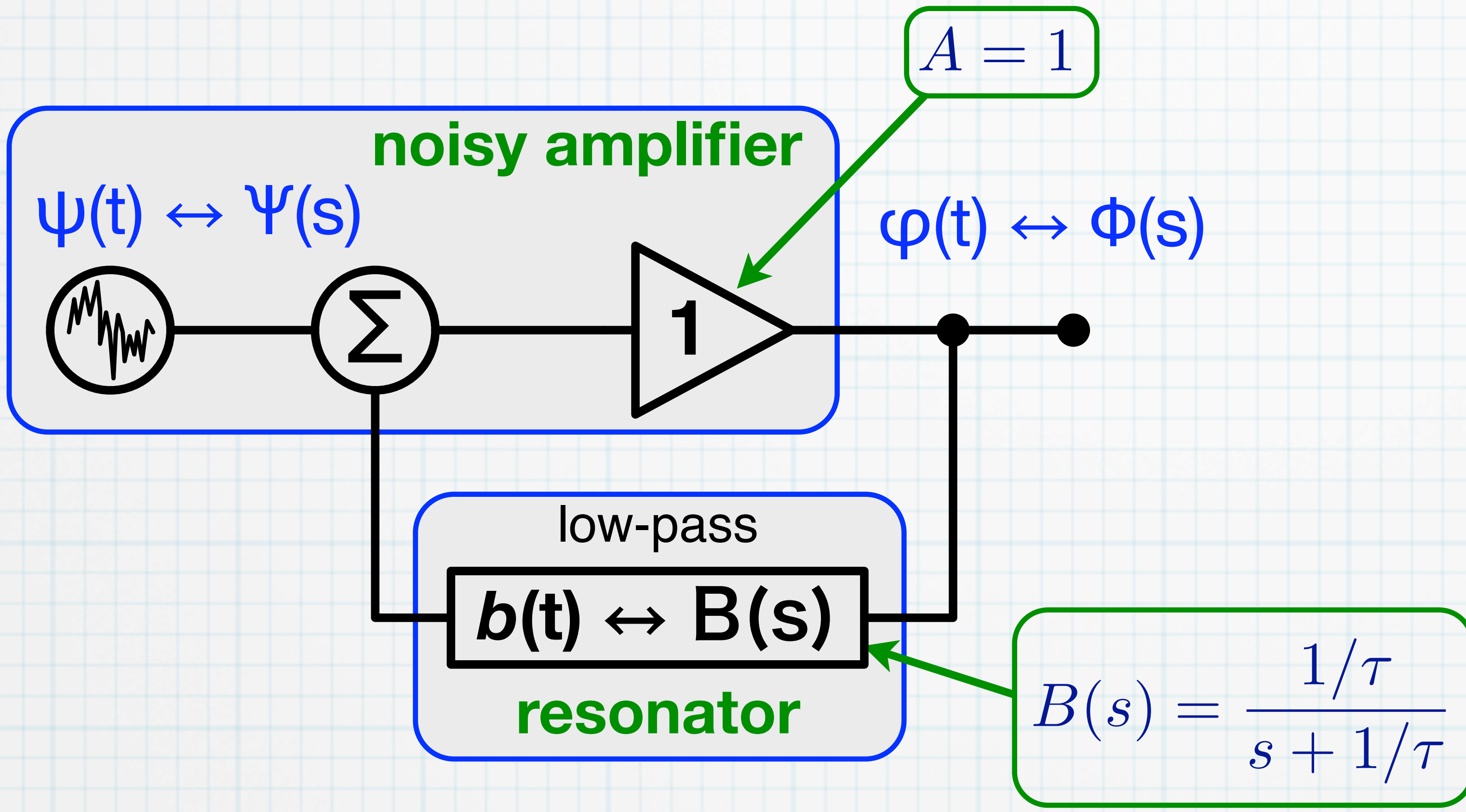




The amplifier
- "copies" the input phase to the out
- adds phase noise

The amplifier
- compresses the amplitude
- adds amplitude noise

Leeson effect

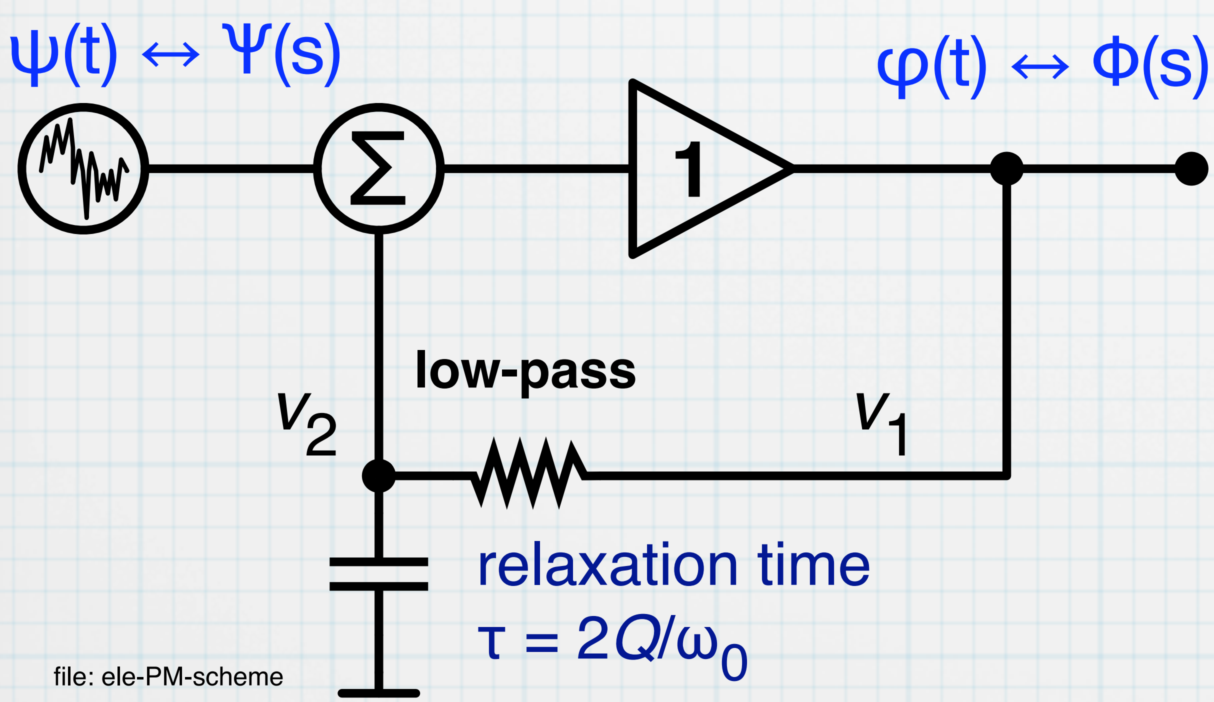


phase-noise transfer function

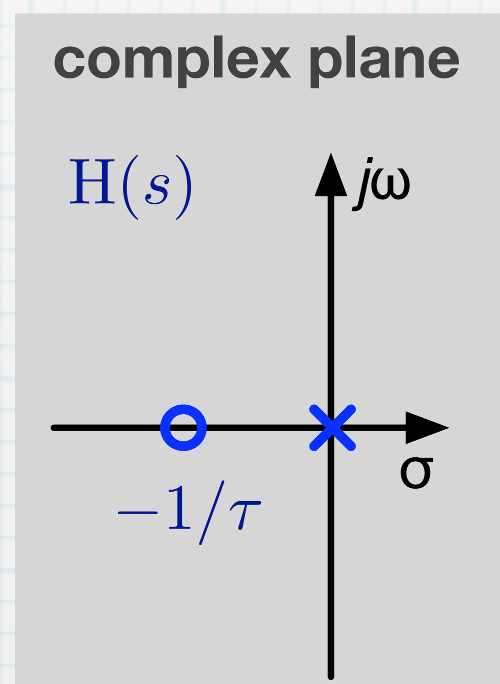
$$\mathrm{H}(s) = \frac{\Phi(s)}{\Psi(s)}$$ **definition**

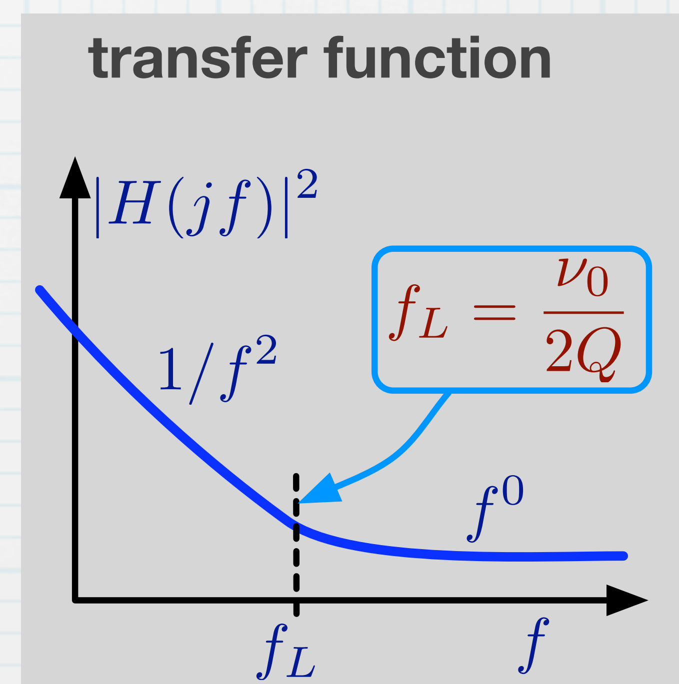
$$\mathrm{H}(s) = \frac{1}{1 + AB(s)}$$ **general feedback theory**

$$\mathrm{H}(s) = \frac{1 + s\tau}{s\tau}$$ **Leeson effect**

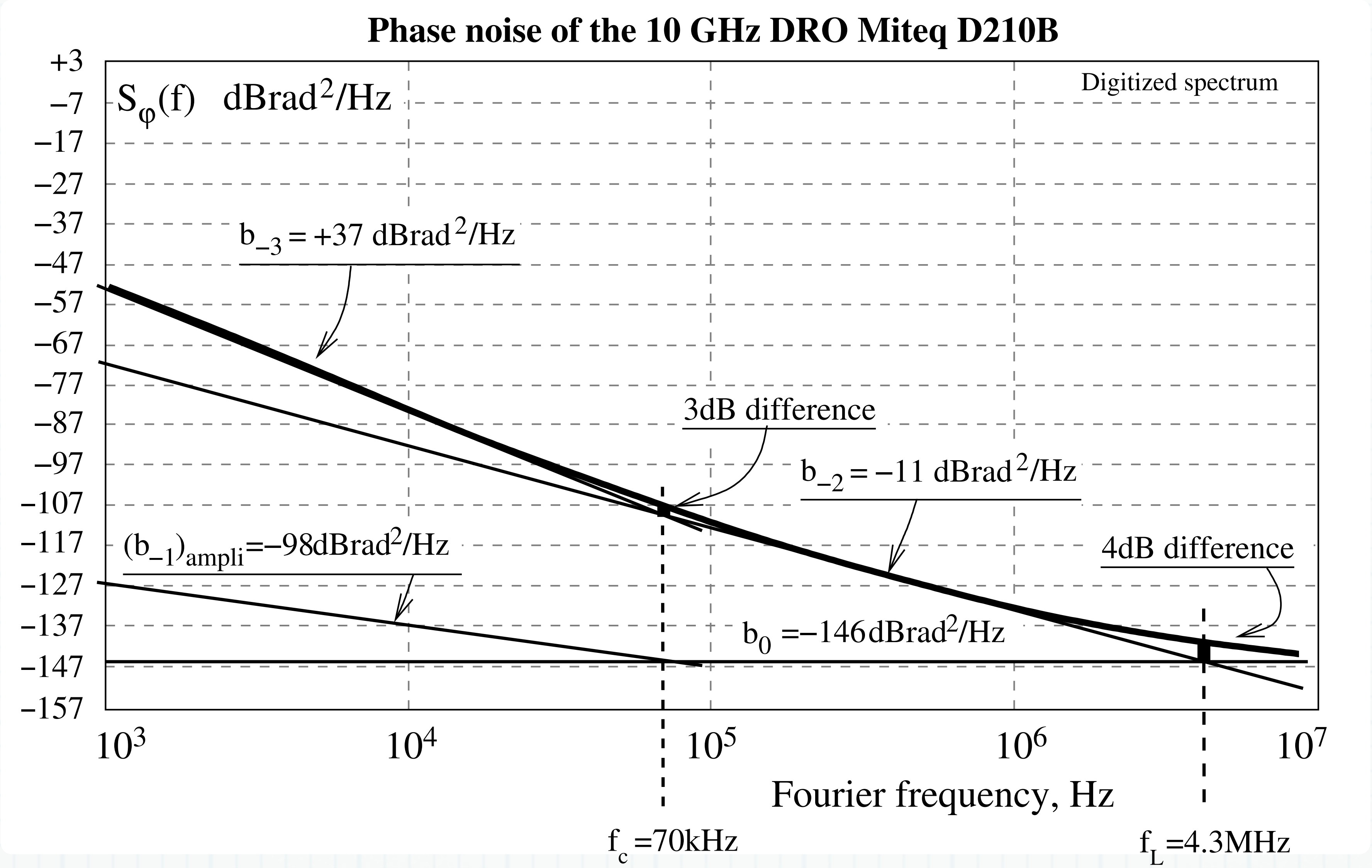


file: ele-PM-scheme





Miteq D210B, 10 GHz DRO



From the table

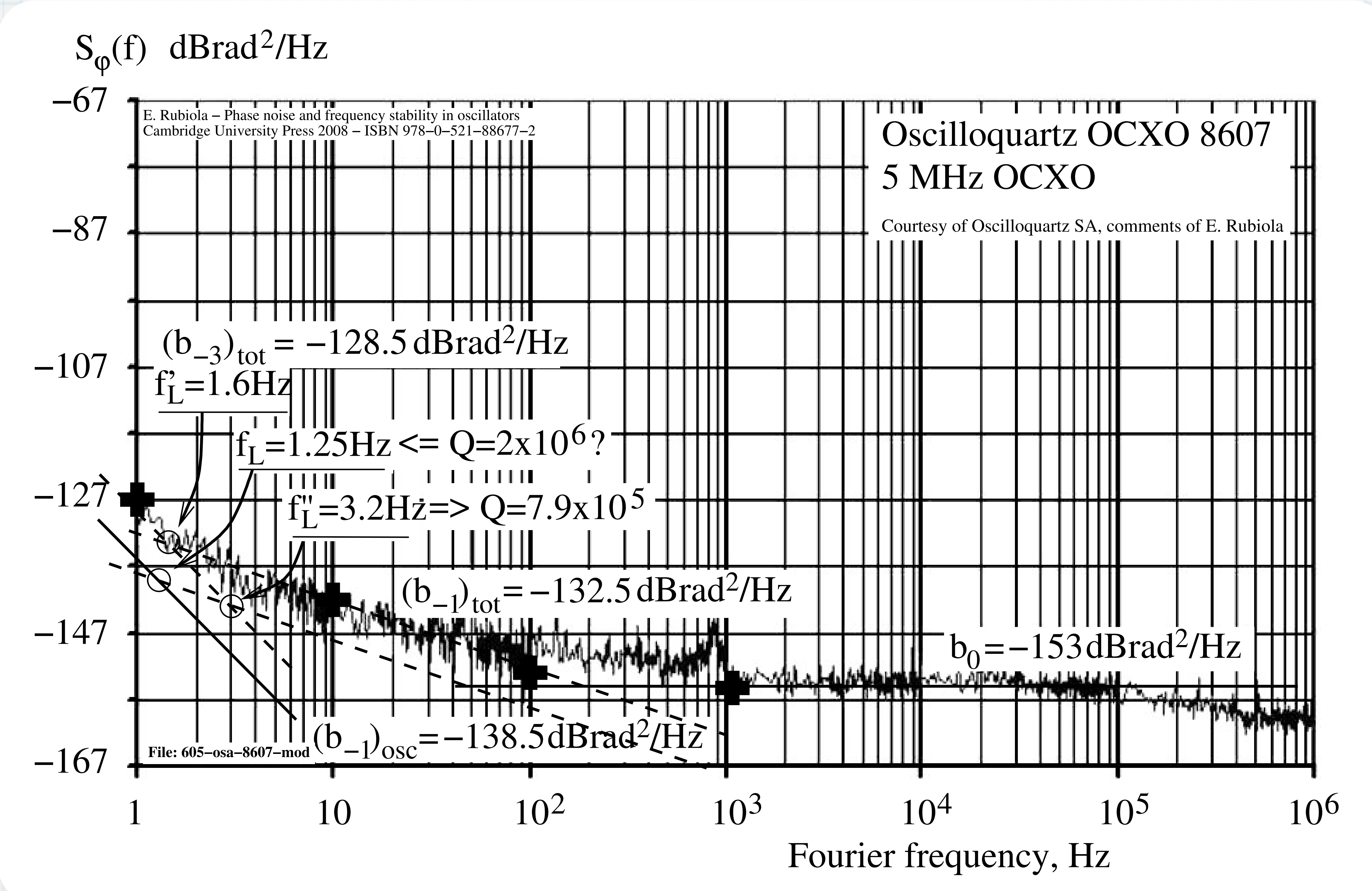
$\sigma^2_y = h_0/2\tau + 2\ln(2) h_{-1}$

$h_0 = b_{-2}/\nu^2_0$

$h_{-1} = b_{-3}/\nu^2_0$

- **$kT_0 = 4\times10^{-21}$ W/Hz (–174 dBm/Hz)**
- **floor –146 dBrad²/Hz, guess F = 1.25 (1 dB) => $P_0 = 2$ µW (–27 dBm)**
- **$f_L = 4.3$ MHz, $f_L = \nu_0/2Q$ => Q = 1160**
- **$f_c = 70$ kHz, $b_{-1}/f = b_0$ => $b_{-1} = 1.8\times10^{-10}$ (–98 dBrad²/Hz) [sust.ampli]**
- **$h_0 = 7.9\times10^{-22}$ and $h_{-1} = 5\times10^{-17}$ => $\sigma_y = 2\times10^{-11}/\sqrt{\tau} + 8.3\times10^{-9}$**

Example – Oscilloquartz 8607



F=1dB b_0 => P_0=–20 dBm

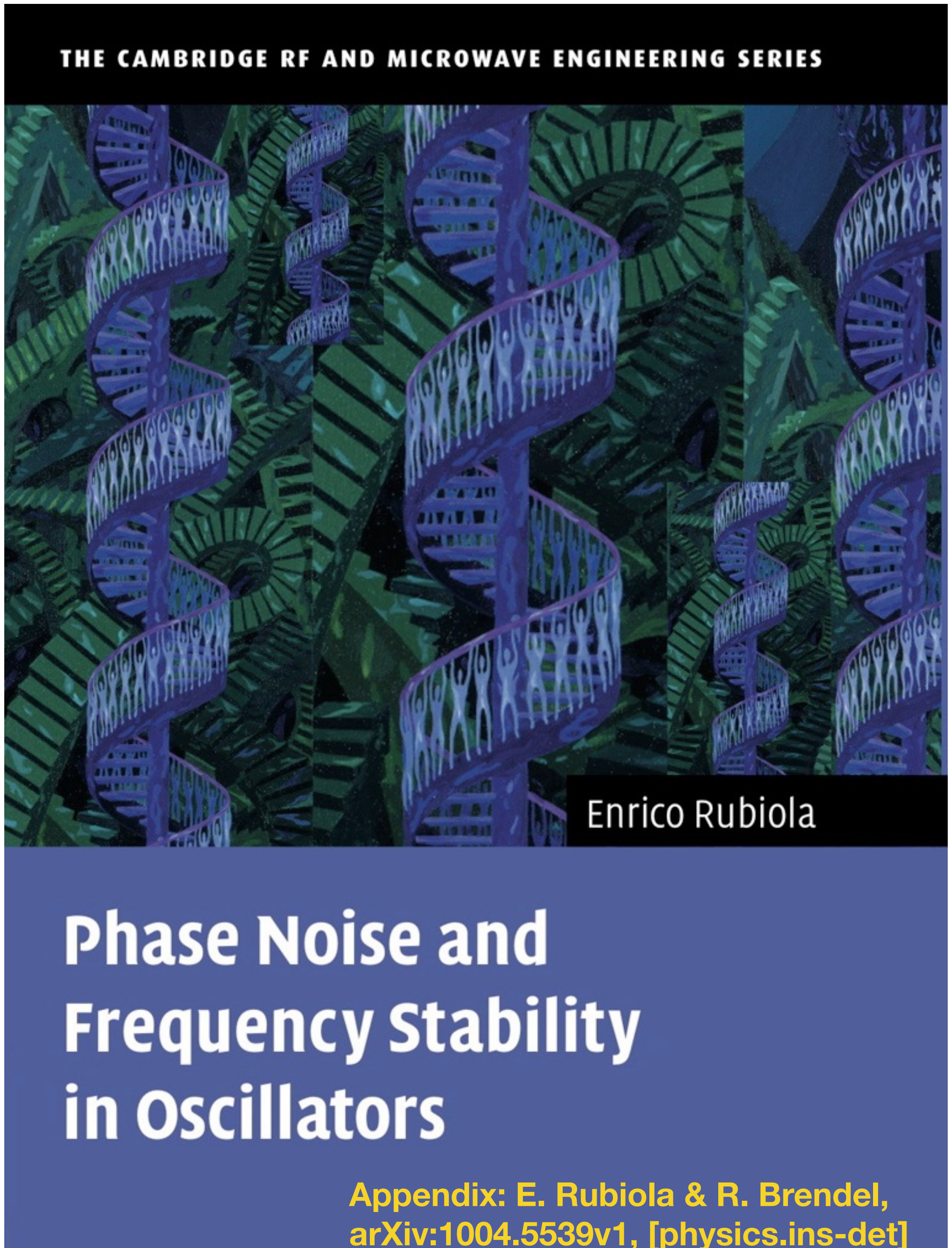
$(b_{-3})_{osc}$ => σ_y=8.8x10^{-14}, Q=7.8x10^5 (too low)

Q $\stackrel{?}{=}$ 2x10^6 => σ_y=3.5x10^{-14} Leeson (too low)

Phase noise and frequency stability in oscillators

Cambridge University Press, 2008

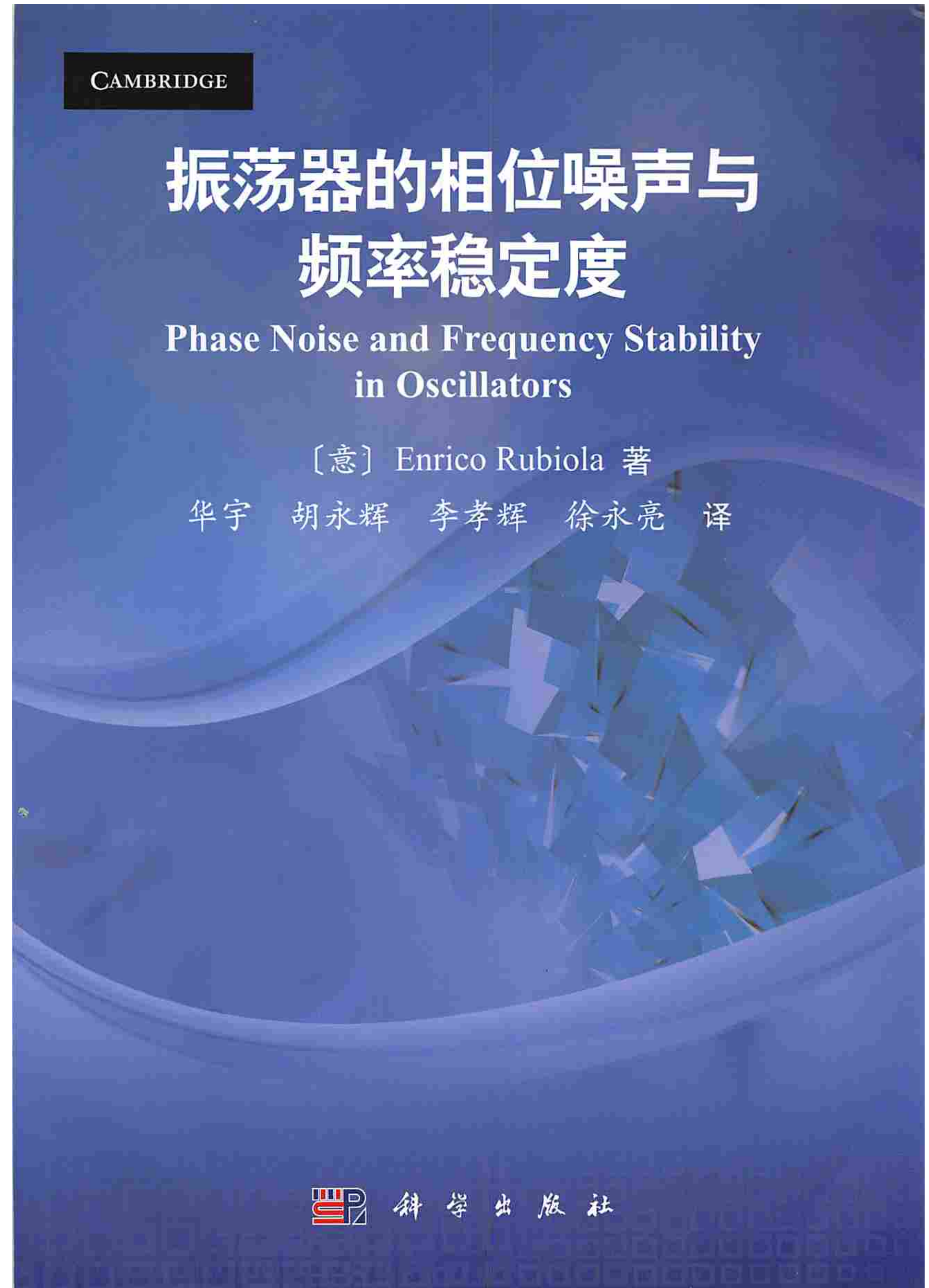
ISBN 978-0-521-88677-2, 978-0-521-15328-7, 978-1-139-23940-0



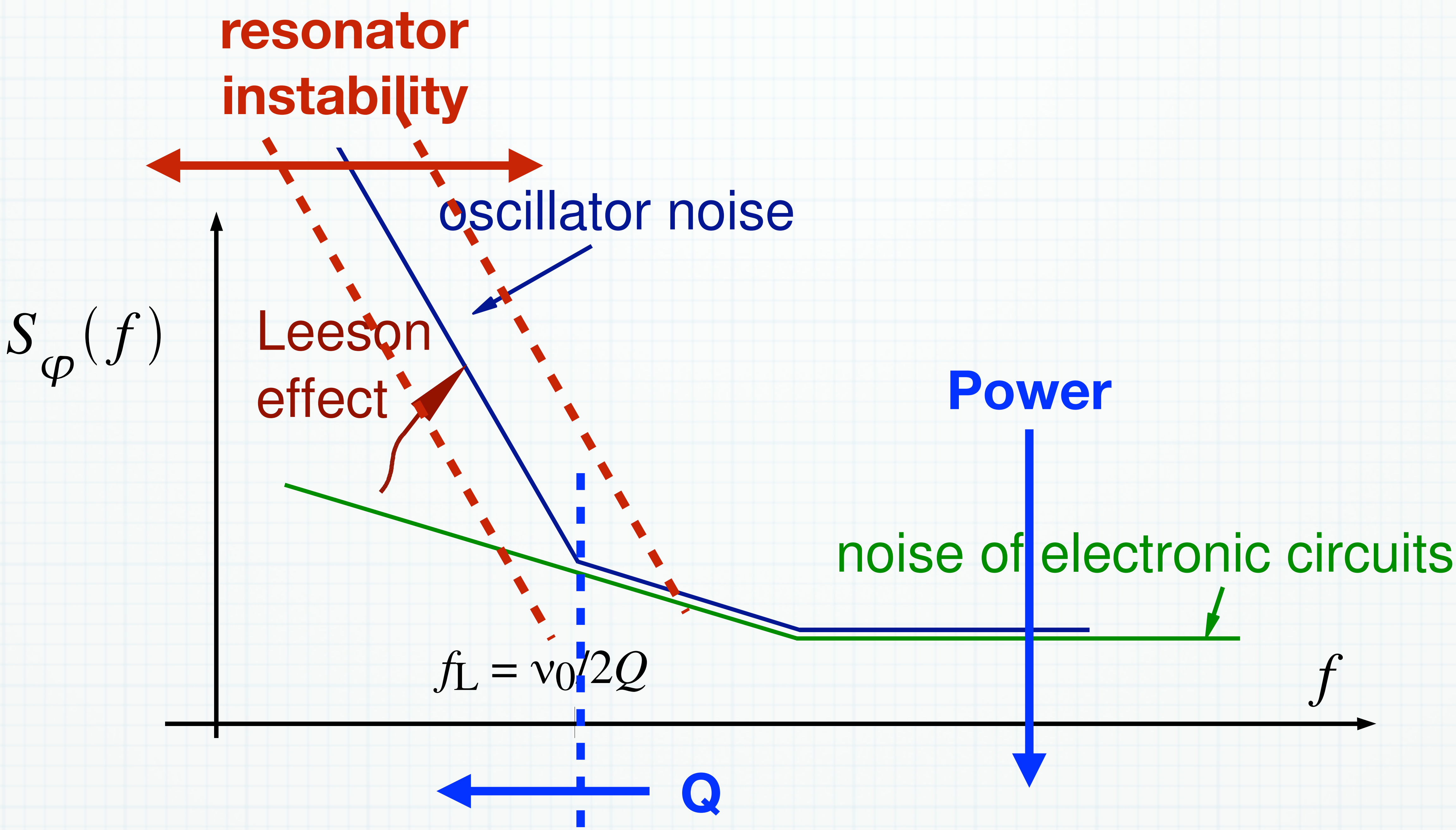
Appendix: E. Rubiola & R. Brendel, arXiv:1004.5539v1, [physics.ins-det]

Simplified Chinese, 2014

ISBN 978-7-03-041231-7

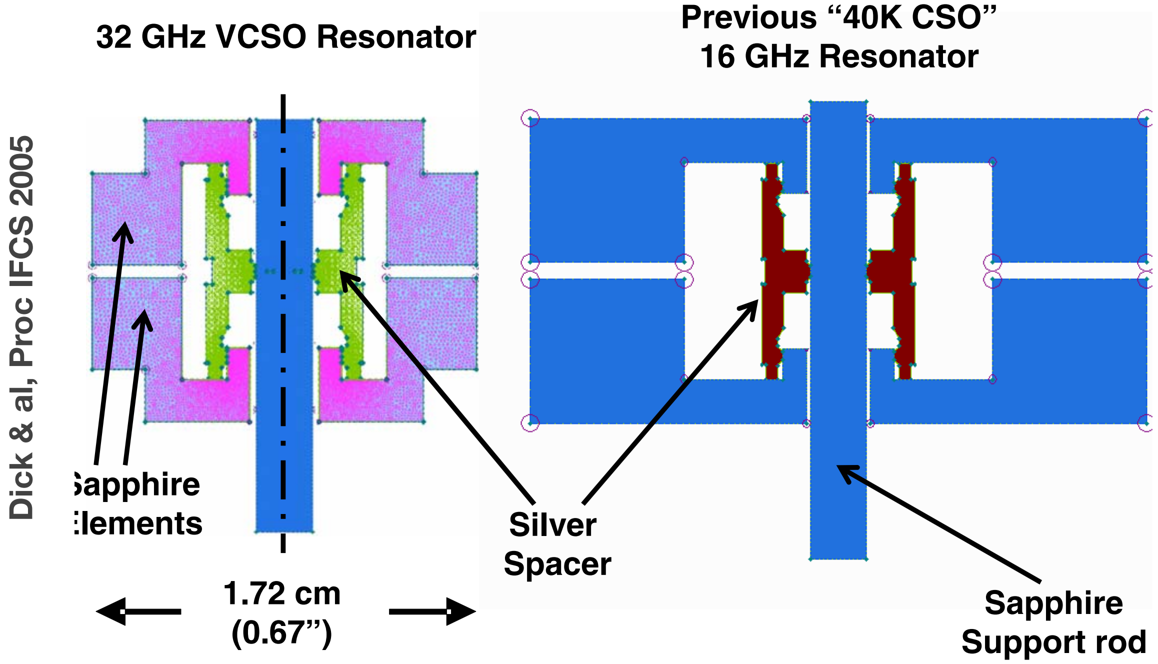


Noise Tradeoff in Oscillators



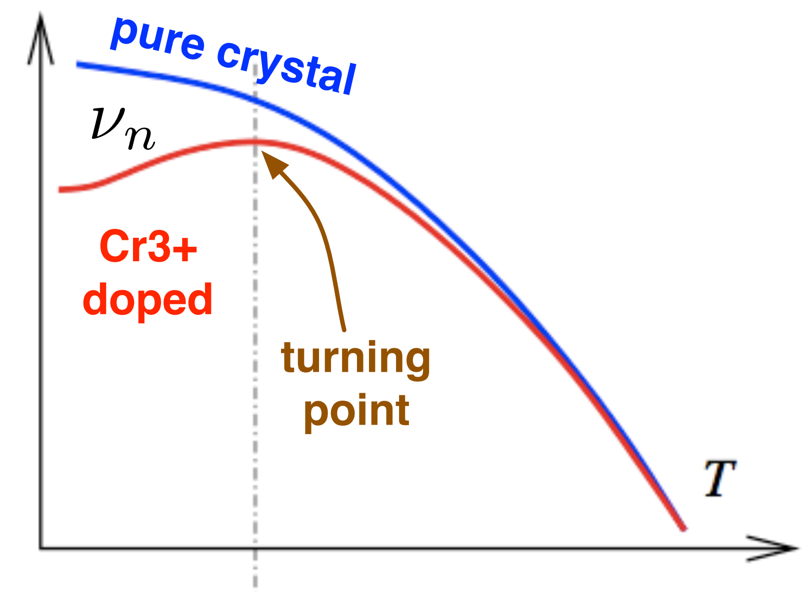
Thermal Compensation – Examples

Thermo-mechanical



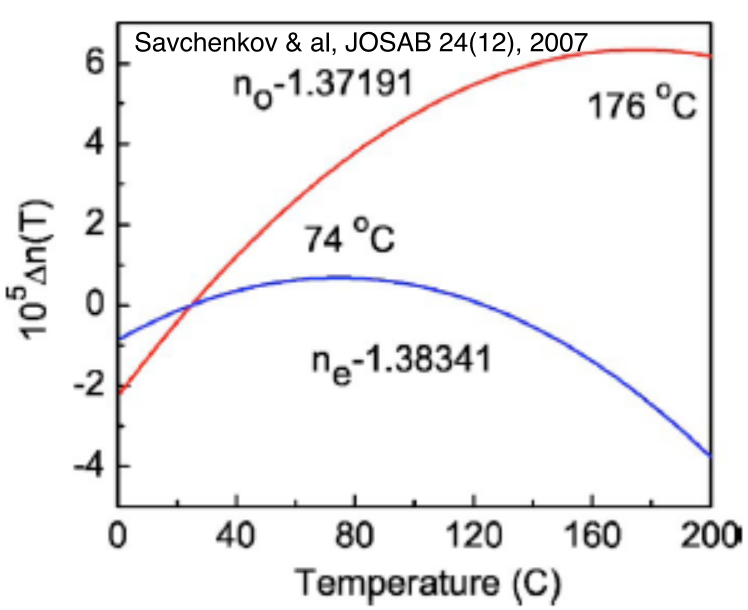
JPL Sapphire (J.Dick)
Derived from the old Lampkin oscillator

Paramagnetic



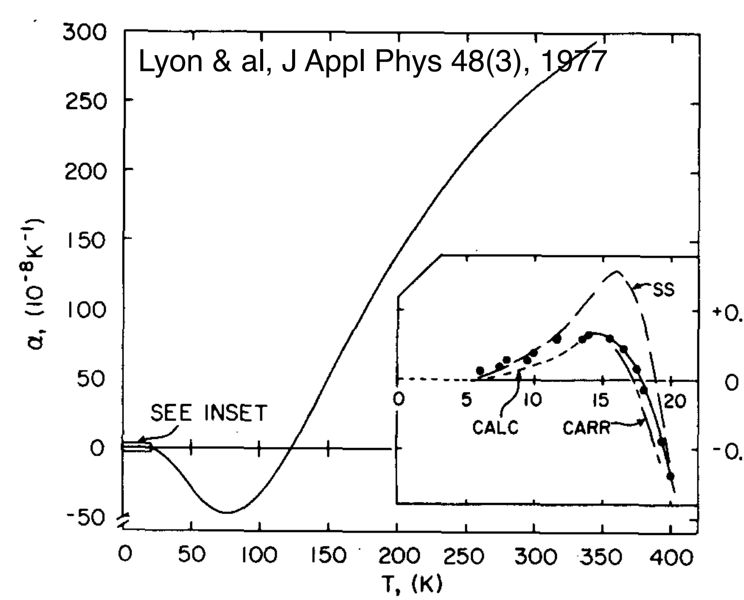
Sapphire Cr3+ impurities @ 6K (V.Giordano / M.Tobar)
Also rutile/sapphire compound @ 80 K (V.Giordano)

Natural – Refraction index



MgF2 whispering gallery (A. Savchenkov)

Natural – Thermal expansion



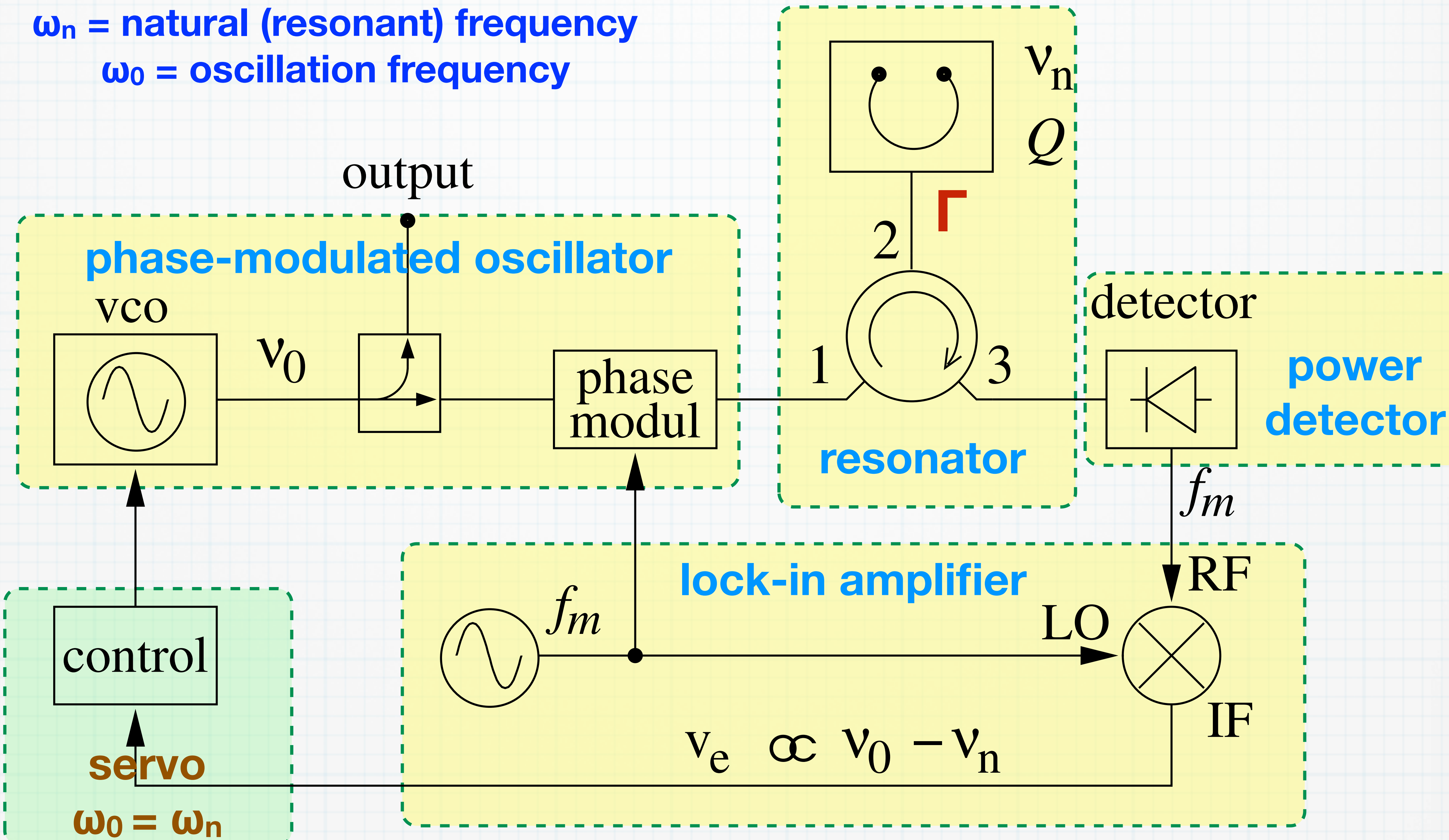
Semiconductor-grade Si @ 124 K (PTB)
@ 17 K (In progress)

And also natural, elastic constant (quartz)

Kessler & al, ArXiv 1112.3854, 2012

The Pound Scheme

ω_n = natural (resonant) frequency
ω_0 = oscillation frequency

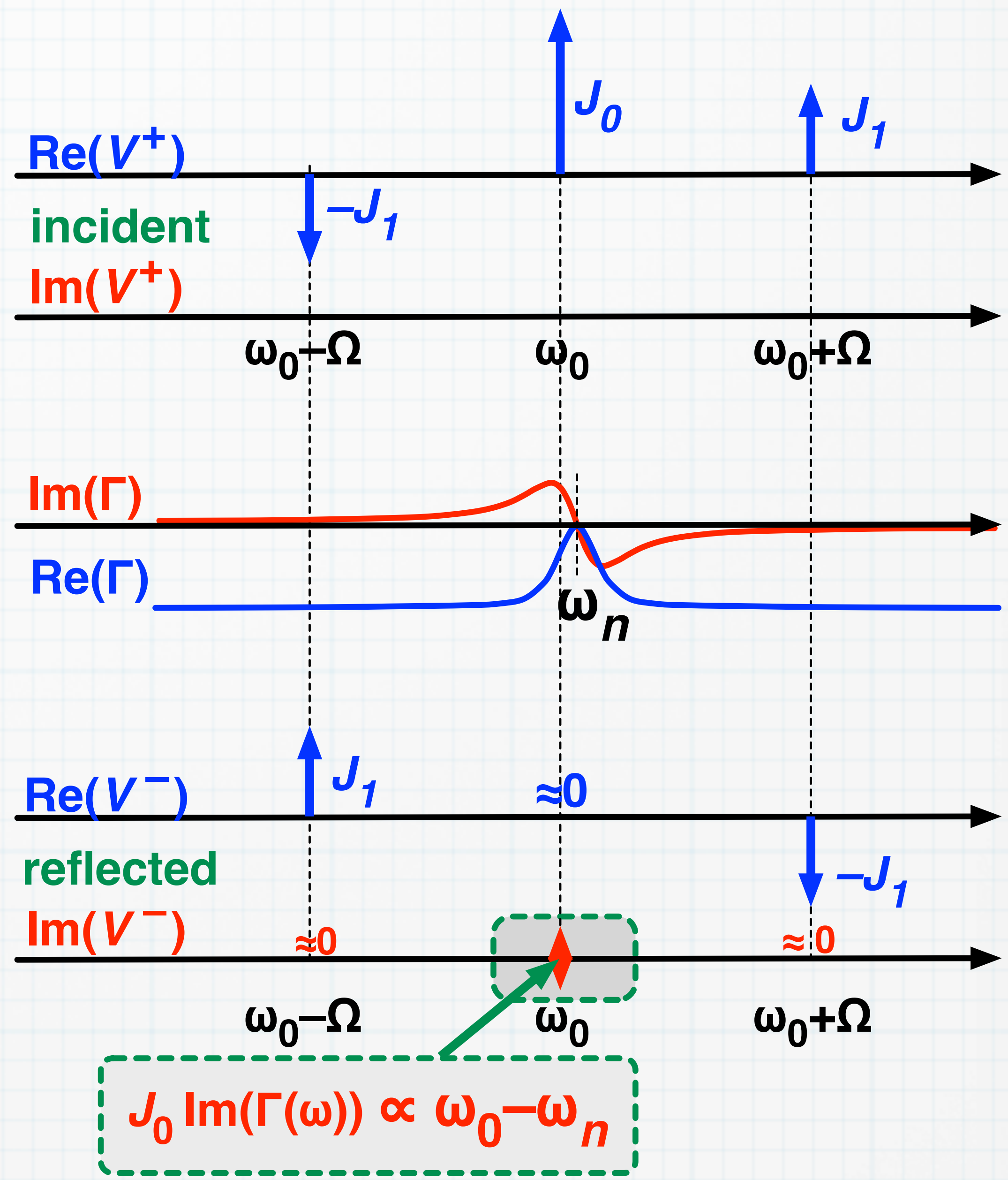


The error signal is proportional to the frequency error

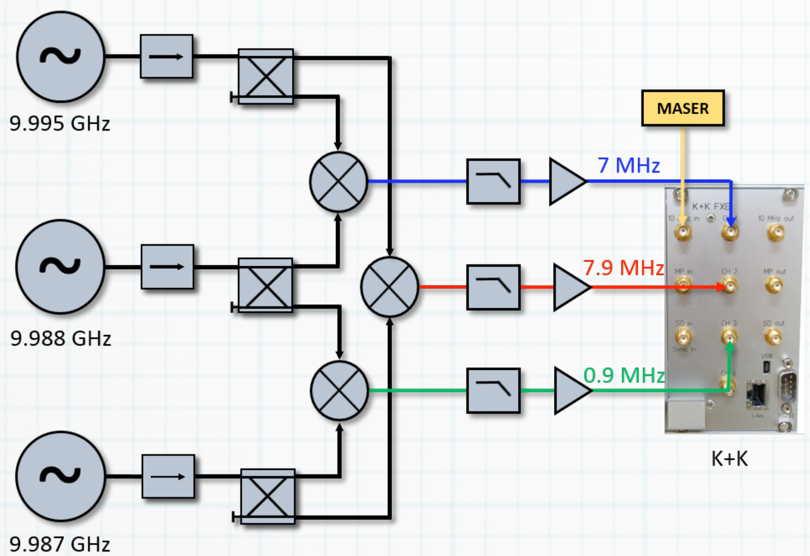
$$v_e = D(\omega_0 - \omega_n)$$

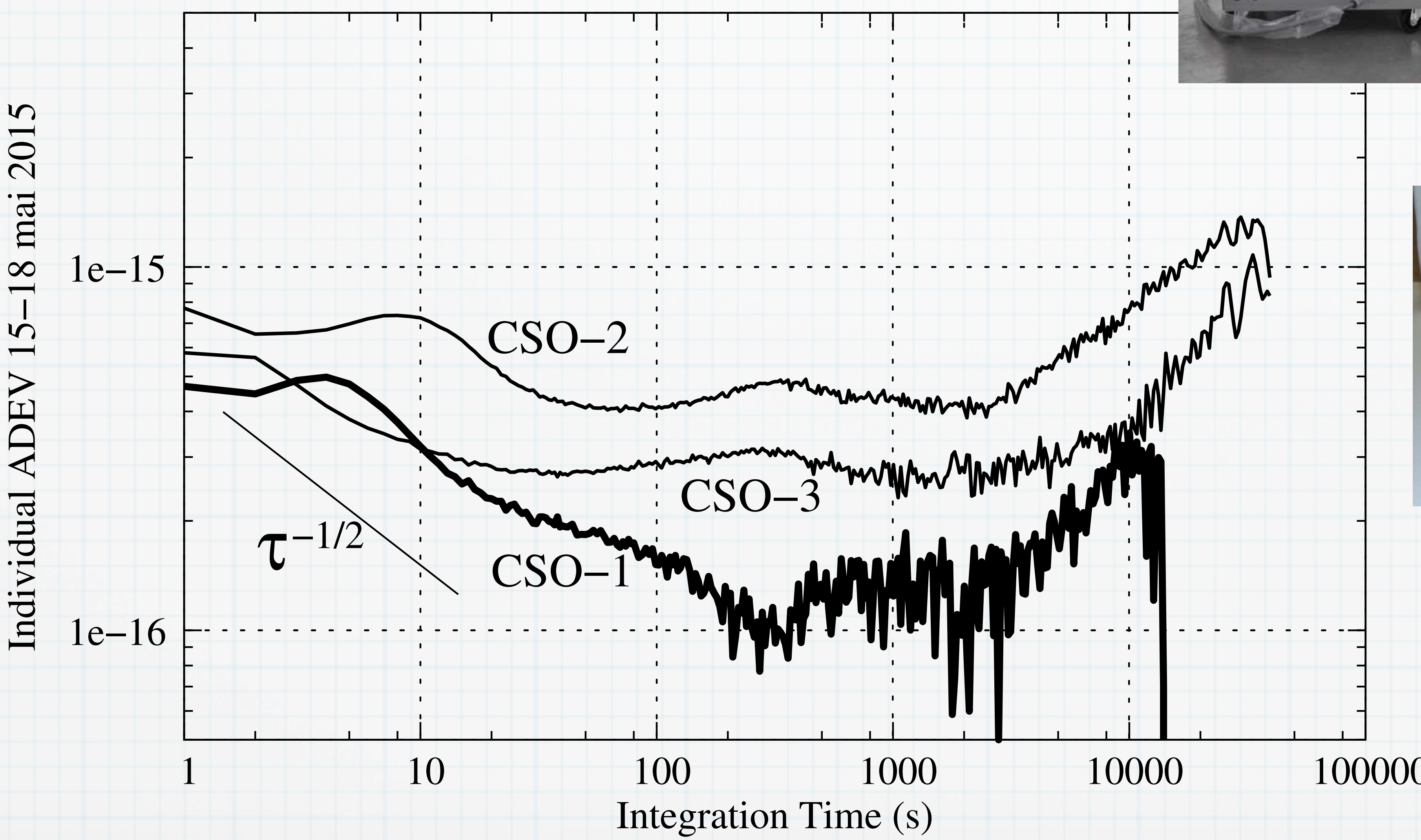
Null Measurement of Im(Γ)

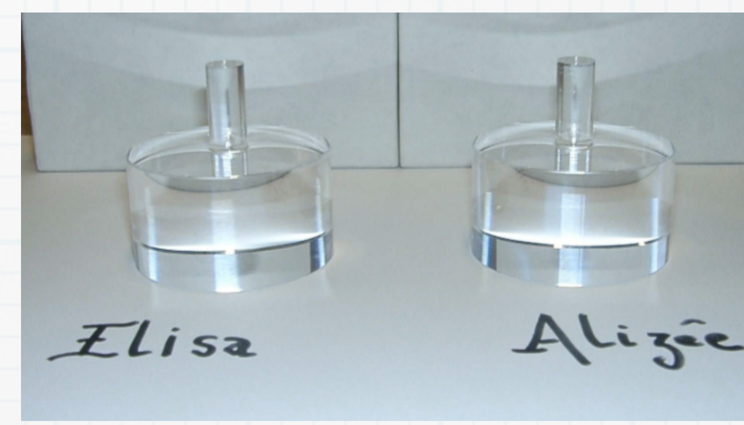
- **Absolute measurements** rely on the "brute force" of instrument accuracy
- **Differential measurements** rely on the difference of two nearly equal quantities, something like $q_2 - q_1$. However similar, this is not our case!
- **Null measurements** rely on the measurement of a quantity as close as possible to zero – ideally zero.
- **The Pound scheme detects**
  - **Null of Im(Γ(ω))**
  - **AC regime, after down-converting to Ω**



Cryogenic Sapphire Oscillator







Selected Oscillators' Personality

- **Quartz**
  - Small, reliable, >25y MTBF
    - 5 MHz: High floor (–155 dBc) and high stability 1E-11 at 1 day, 1E–13 ADEV floor
    - 100 MHz: Low floor (–180 dBc), fair stability
- **YIG (10 GHz)**
  - Low noise at high frequency (-160 dBc), but unstable
- **DRO (10 GHz)**
  - Low noise at high frequency (-160 dBc)
  - No inherent thermal compensation
- **Sapphire (10 GHz)**
  - 300 K: Q=2E5, TC=70ppm/K, low floor (–180 dBc)
  - ≈77 K: Q=3E7, TC≈few-ppm/K, low floor (–180 dBc)
  - 5 K Q>1E9, TC=0,
    - w/o Pound control: Low floor (–180 dBc possible)
    - with Pound: <1E-15 at 1 s, parts-E-16, 2E-15 at 1 d

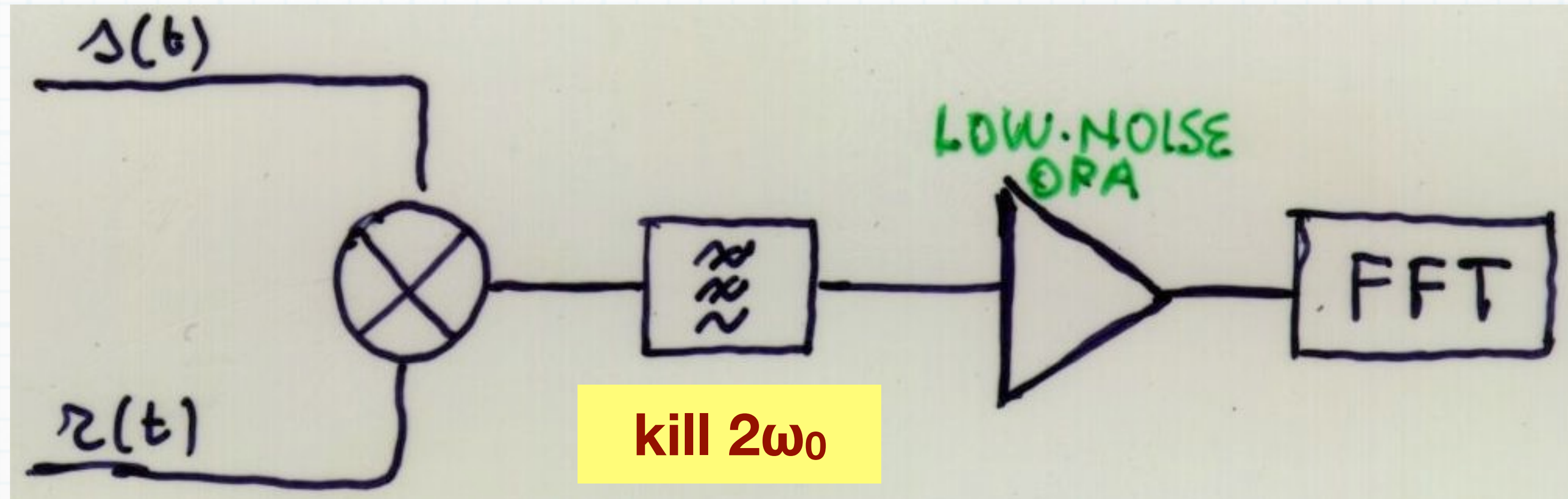
Measurement Methods

Double Balanced Mixer

saturated multiplier => phase-to-voltage detector $v_o(t) = k_\varphi\, \varphi(t)$

$\cos(\omega_0 t+\varphi)$

$\sin(\omega_0 t)$



1 – Power

narrow power range:
±5 dB around P_{nom} = 7–13 dBm
r(t) and s(t) should have ~ same P

2 – Flicker noise

due to the mixer internal diodes
typical S_φ = –140 dBrad²/Hz at 1 Hz
in average-good conditions

3 – Low gain

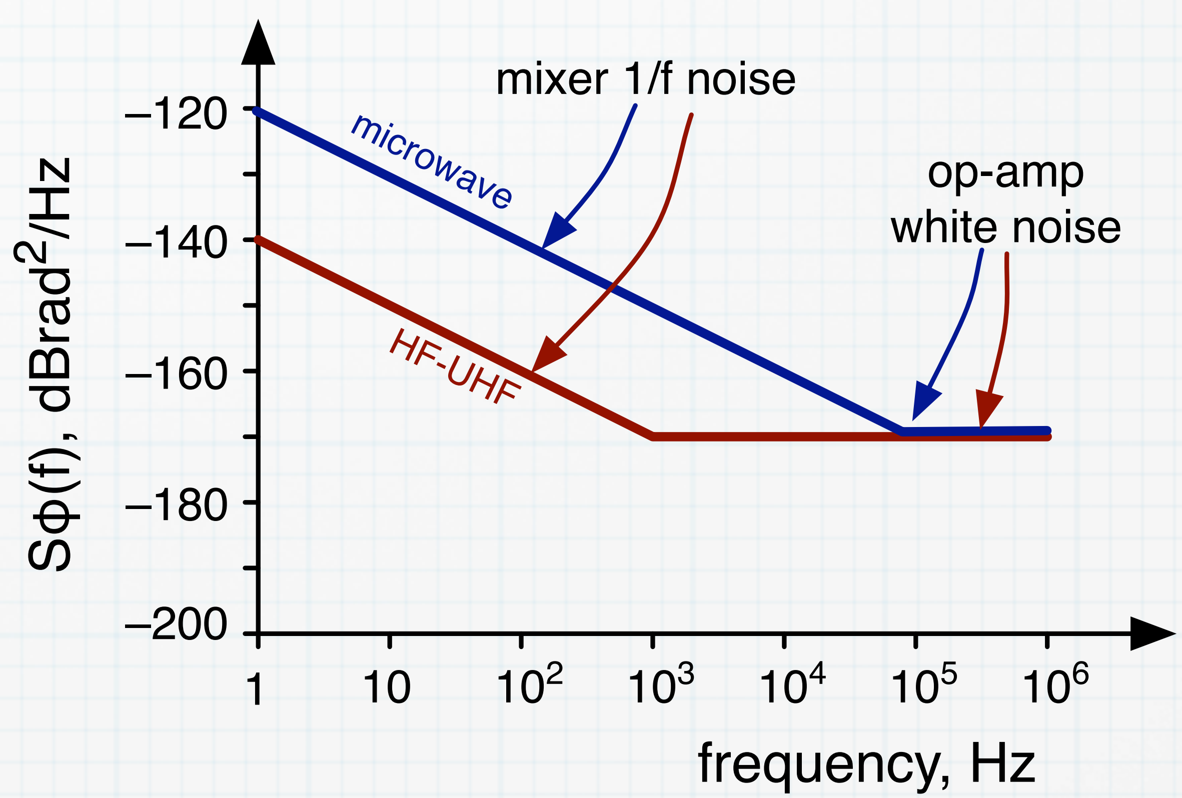
k_φ ~ 0.2–0.3 V/rad typ.
–10 to –14 dBV/rad

4 – White noise <=> operational amplifier

5 – Takes in noise <=> power-to-offset conversion

6 – High sensitivity to 50 Hz magnetic field

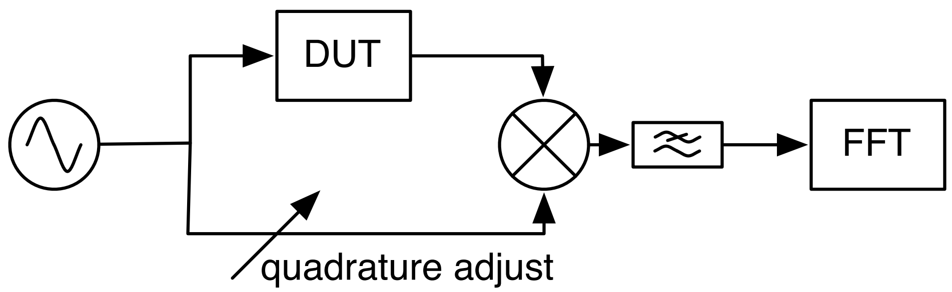
mixer background noise



E. Rubiola, *Tutorial on the double-balanced mixer*, arXiv/physics/0608211,

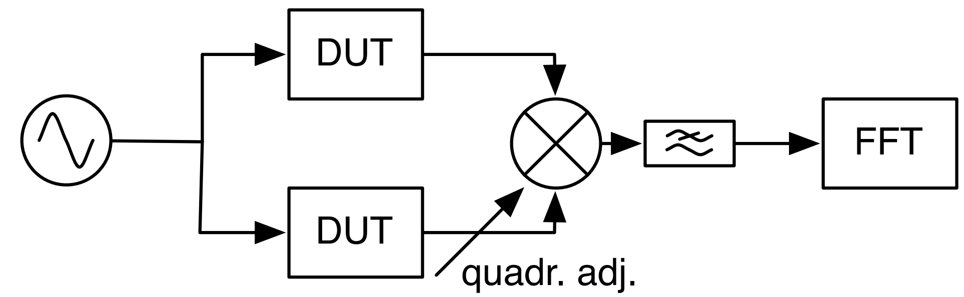
Useful Schemes

two-port device under test

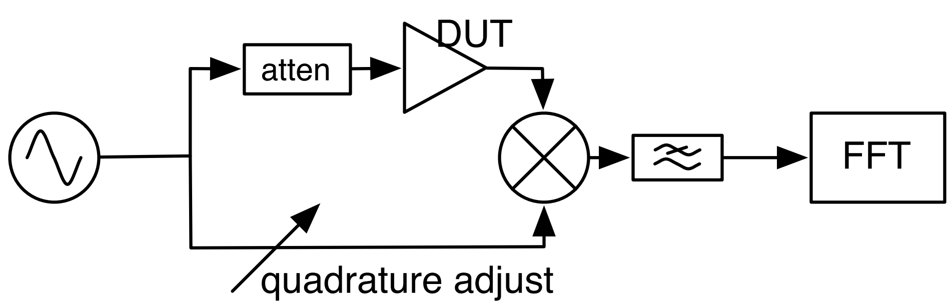


a pair of two-port devices

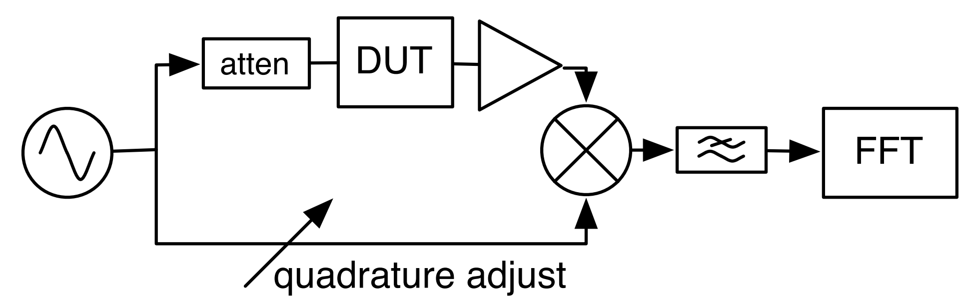
3 dB improved sensitivity



the measurement of an amplifier needs an attenuator

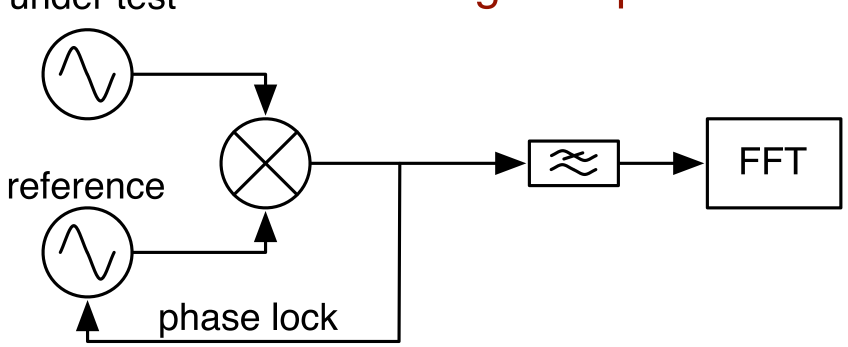


the measurement of a low-power DUT needs an amplifier, which flickers

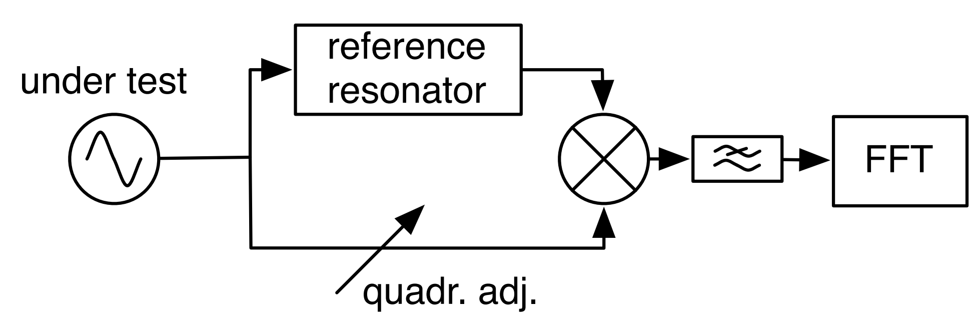


measure two oscillators

best use a tight loop

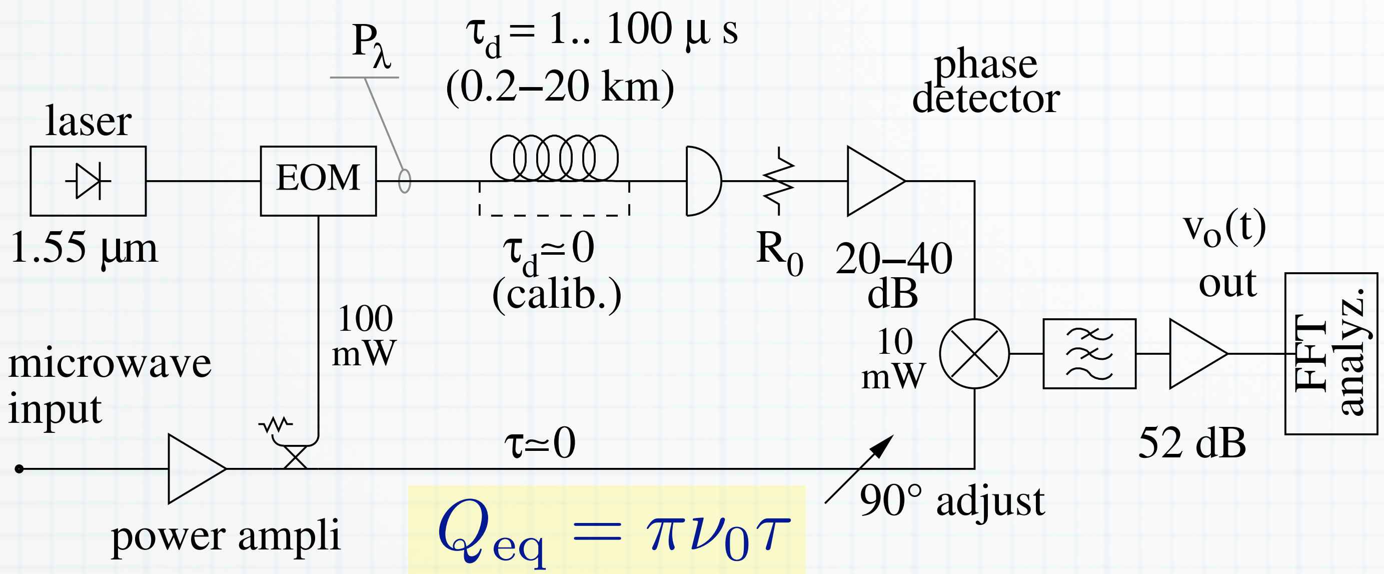


measure an oscillator vs. a resonator



Opto-Electronic Discriminator

Rubiola, Salik, Huang, Yu, Maleki, JOSA-B 22(5) p.987–997 (2005)



$$Q_{\text{eq}} = \pi \nu_0 \tau$$

The short arm can be a microwave cable or a photonic channel

Laplace transforms

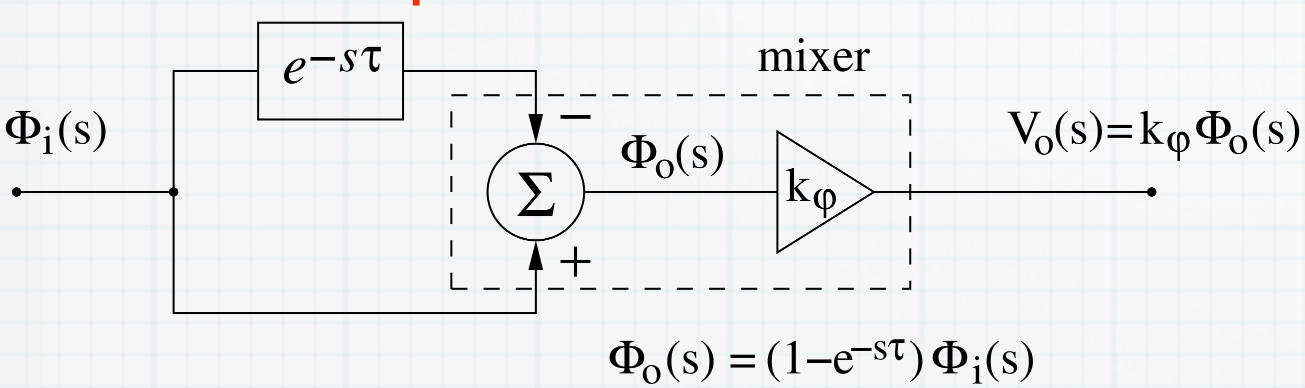
$$\Phi(s) = H_\varphi(s)\Phi_i(s)$$

$$|H_\varphi(f)|^2 = 4\sin^2(\pi f \tau)$$

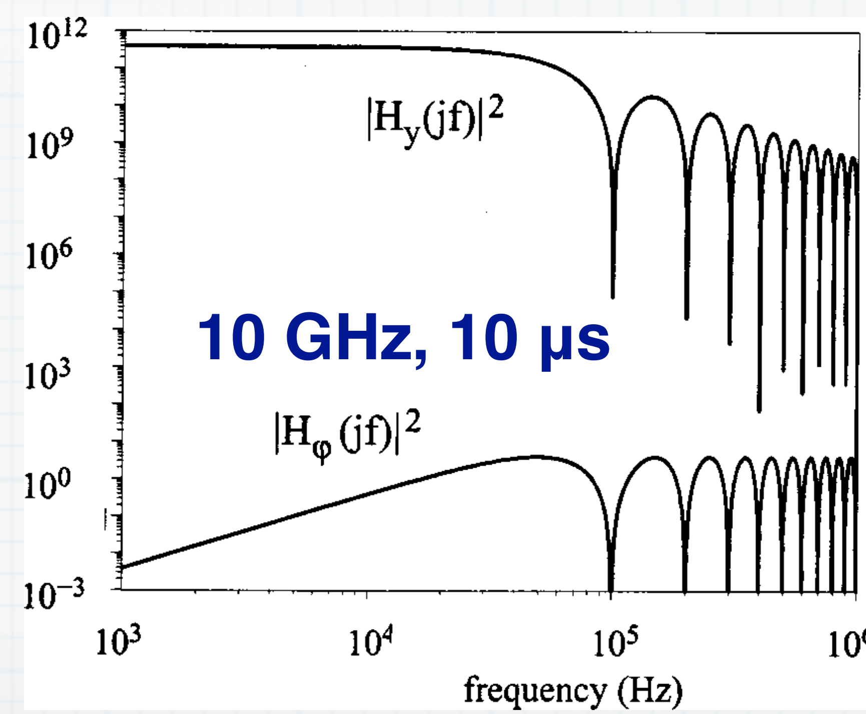
$$S_y(f) = |H_y(f)|^2\, S_{\varphi\, i}(s)$$

$$|H_y(f)|^2 = \frac{4\nu_0^2}{f^2}\,\sin^2(\pi f \tau)$$

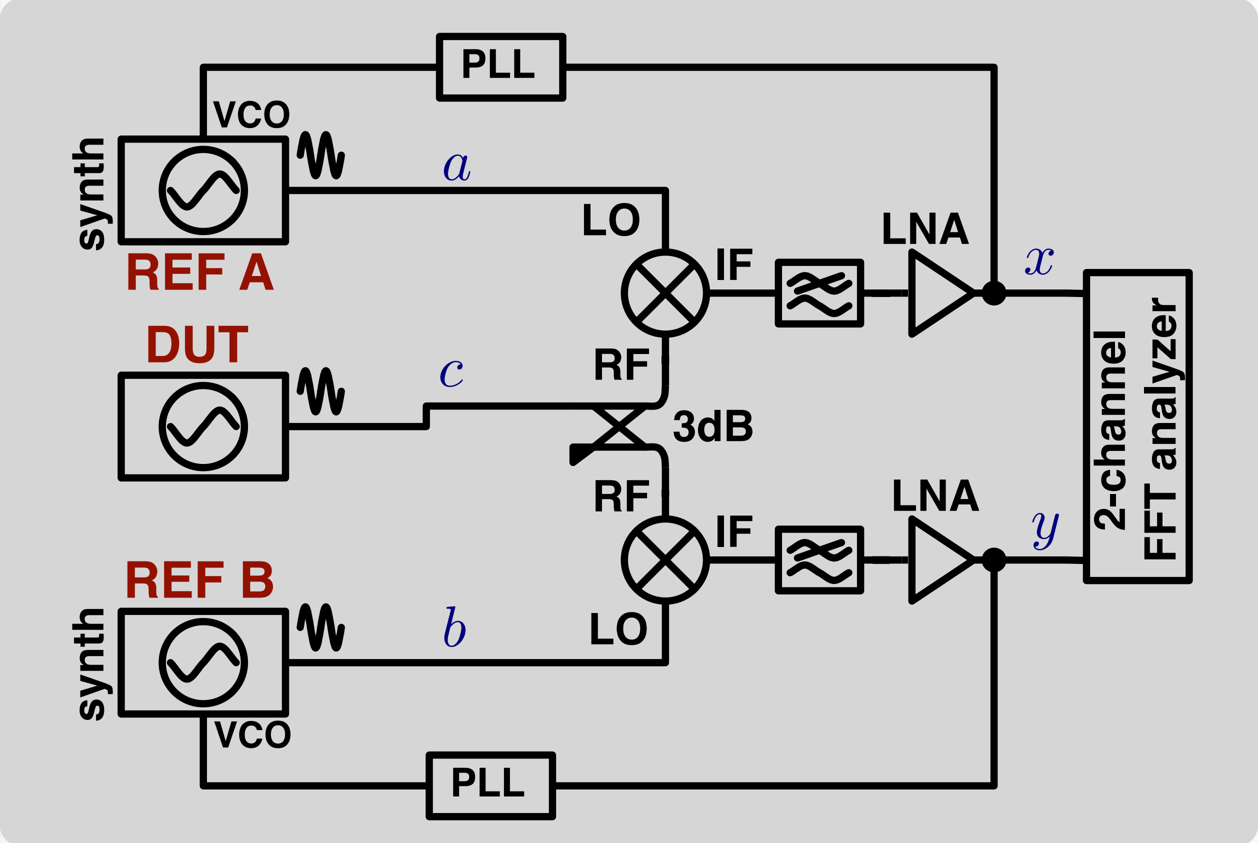
Laplace transforms



- delay –> frequency-to-phase conversion
- works at any frequency
- long delay (microseconds) is necessary for high sensitivity
- the delay line must be an optical fiber
  fiber: attenuation 0.2 dB/km, thermal coeff. 6.8 10^{-6}/K
  cable: attenuation 0.8 dB/m, thermal coeff. ~ 10^{-3}/K



Cross Spectrum Method



A, B = instrument background
C = DUT noise

Channel 1 X = C – A
Channel 2 Y = C – B

A, B, C are independent
Re{ } and Im{ } are independent

Normalization: in 1 Hz bandwidth var{A} = var{B} = 1, var{C}=κ^2
var{A'} = var{A"} = var{B'} = var{B"} = 1/2, and var{C'} = var{C"} = $\kappa^2/2$

Cross spectrum

$$\langle S_{yx}\rangle_m = \frac{1}{T}\langle YX^*\rangle_m = \frac{1}{T}\langle (Y'+iY'')\times(X'-iX'')\rangle_m$$

Use

$$X = (C'+iC'')-(A'+iA'') \quad \text{and} \quad Y = (C'+iC'')-(B'+iB'')$$

Split S_{yx} into three sets

$$\langle S_{yx}\rangle_m = \underbrace{\langle S_{yx}\rangle_m\big|_{\text{instr}}}_{\text{background only}} + \underbrace{\langle S_{yx}\rangle_m\big|_{\text{mixed}}}_{\text{background and DUT noise}} + \underbrace{\langle S_{yx}\rangle_m\big|_{\text{DUT}}}_{\text{DUT noise only}}$$

S_{yx} with correlated term κ≠0 (2)

All the DUT signal goes in Re{S_{yx}}, Im{S_{yx}} contains only noise

Real

$$\Re\left\{\langle S_{yx}\rangle_m\right\} = \frac{1}{T}\left\{\langle B'A' + B''A''\rangle_m + \langle B'C' + B''C''\rangle_m + \langle A'C' + A''C''\rangle_m + \langle (C')^2 + (C'')^2\rangle_m\right\}$$

var=1/2 | var=1/2 | var= κ²/2 | var=1/2 | var= κ²/2 | var= κ²/2

avg=0
var=1/4

avg=0
var=κ²/4

avg = κ²
var = κ⁴

avg = 0 | **avg = 0** | **avg = 0**

avg = κ²
var = κ⁴/m

avg = 0, var = (1+2κ²)/2m

Imaginary

$$\Im\left\{\langle S_{yx}\rangle_m\right\} = \frac{1}{T}\left\{\langle B''A' + B'A''\rangle_m + \langle B''C' - B'C''\rangle_m + \langle A'C'' - A''C'\rangle_m\right\}$$

avg=0
var=1/4

avg=0
var=κ²/4

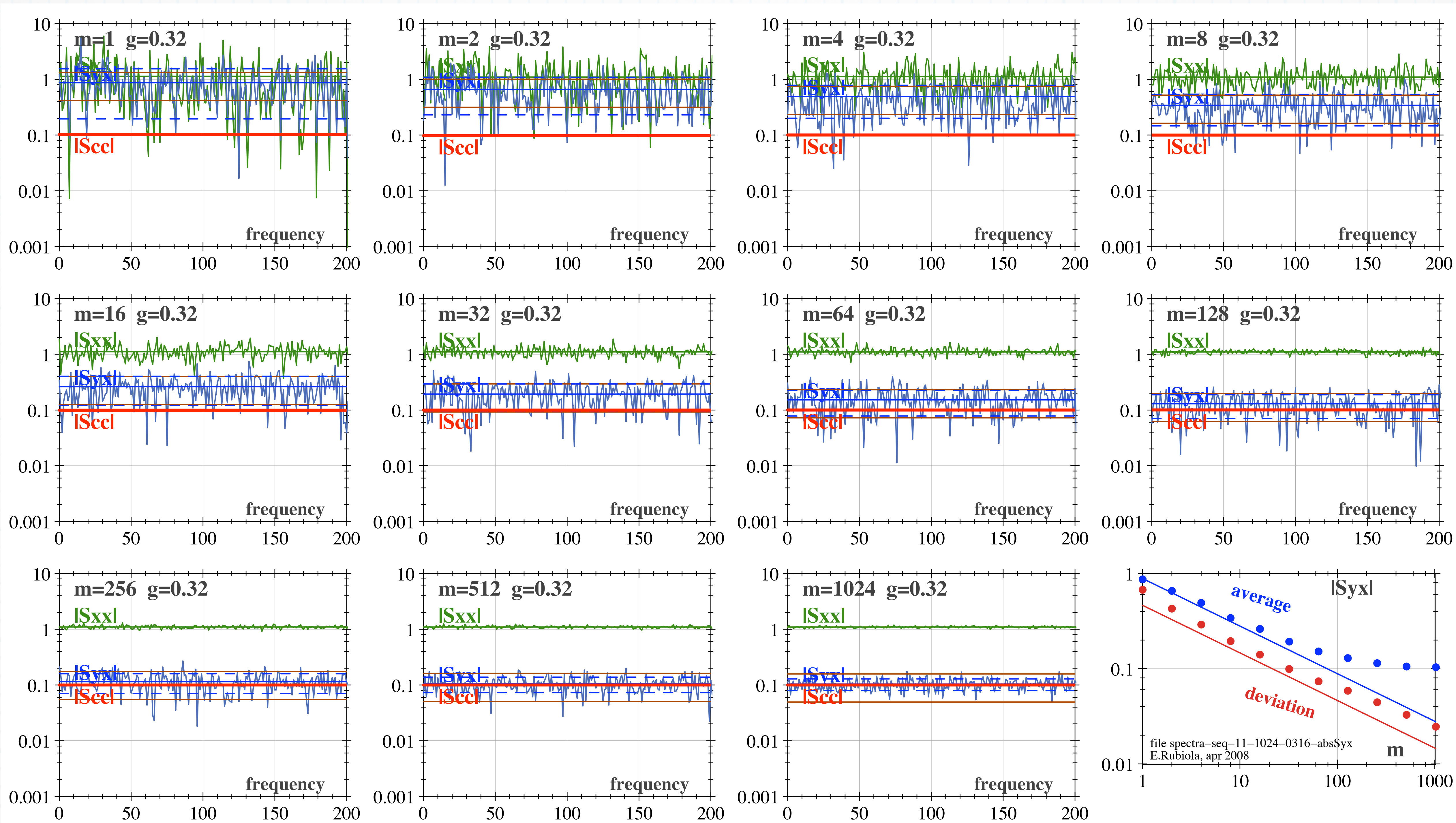
avg = 0 | **avg = 0** | **avg = 0**

avg = 0, var = (1+2κ²)/2m

Normalization: in 1 Hz bandwidth var{A} = var{B} = 1, var{C}=κ²
var{A'} = var{A"} = var{B'} = var{B"} = 1/2, and var{C'} = var{C"} = κ²/2

A, B, C are independent Gaussian noises
Re{ } and Im{ } are independent Gaussian noises

Measurement (C≠0), |Syx|



Running the measurement, m increases

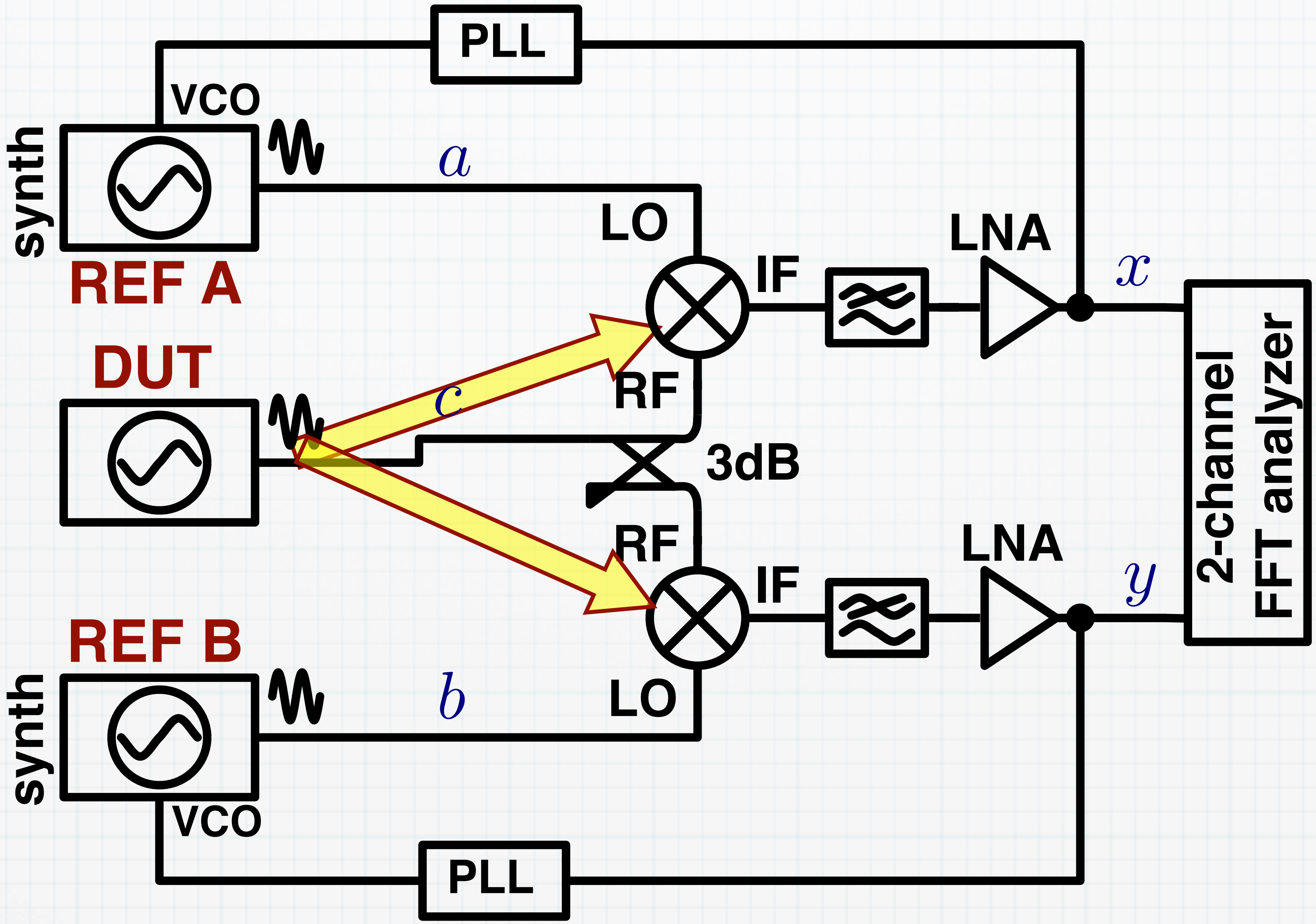
S_{xx} shrinks => better confidence level

S_{yx} decreases => higher single-channel noise rejection

The DUT AM noise is correlated

Power fluctuation ΔP –> ΔV_{os}

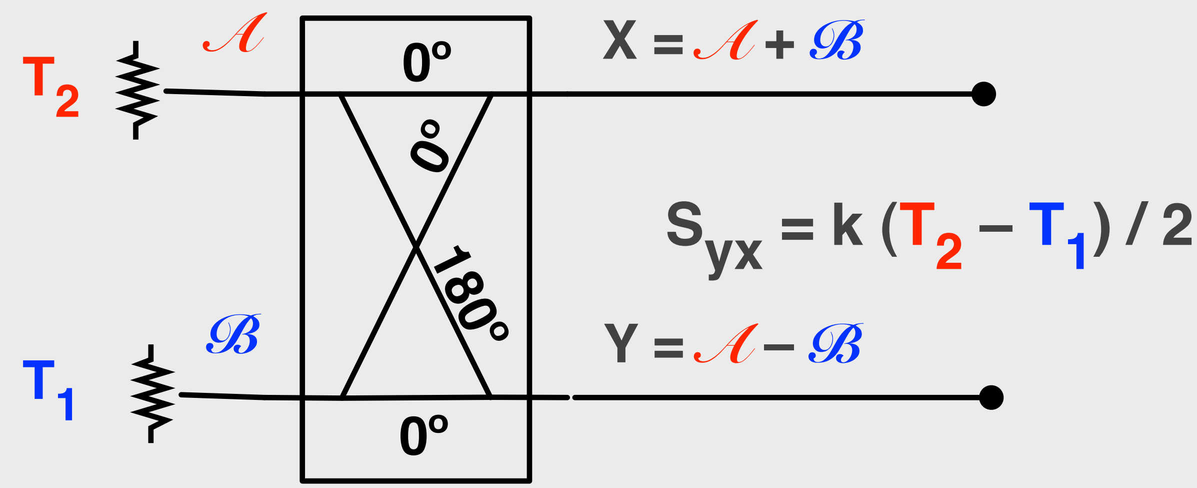
AM noise $S_\alpha(f)$ –> $S_v(f)$, interpreted as $S_\varphi(f)$



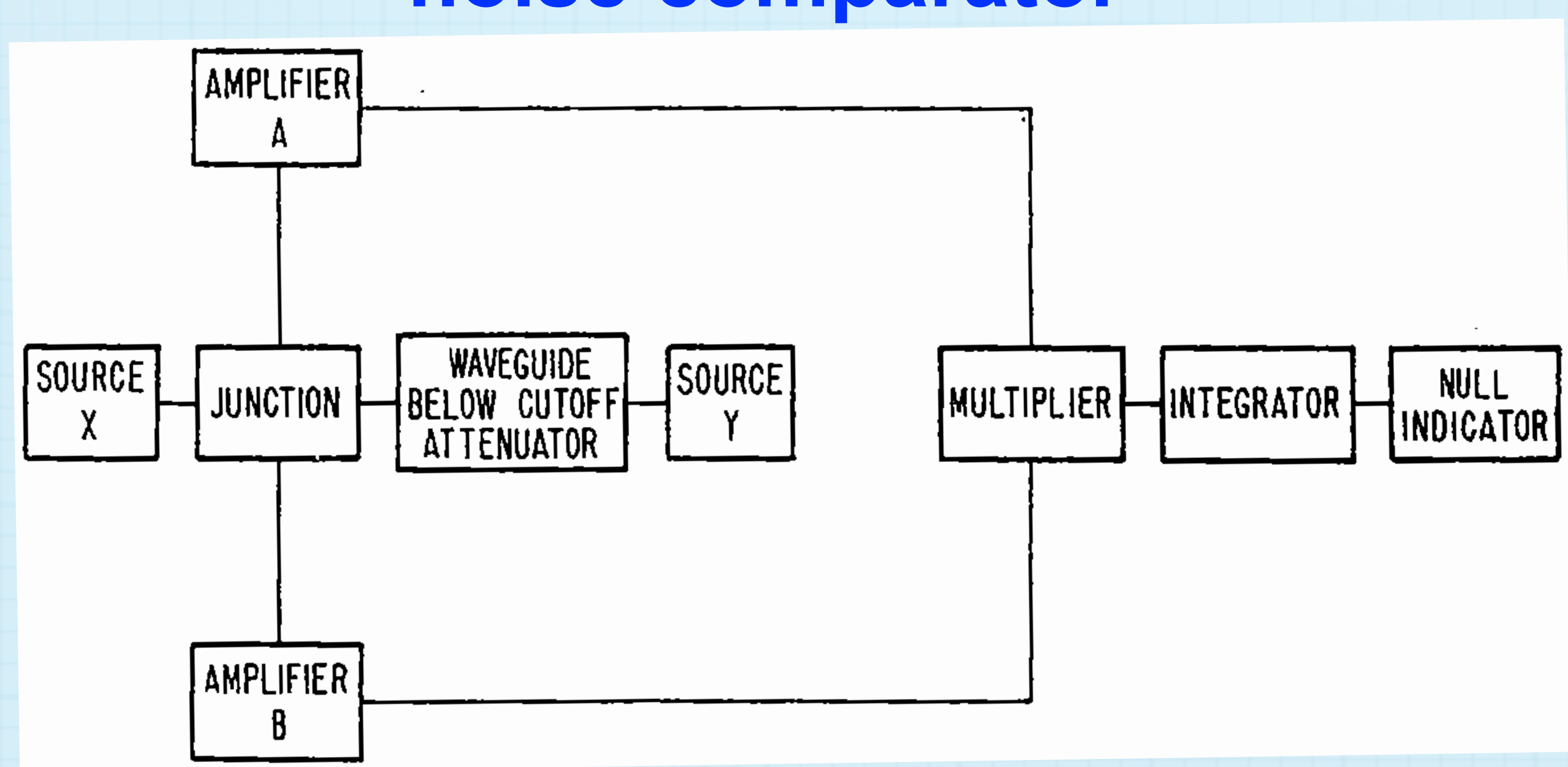
E. Rubiola, R. Boudot, IEEE Transact. UFFC 54(5) pp.926-932, may 2007

Radiometry & Johnson thermometry

correlation and anti-correlation



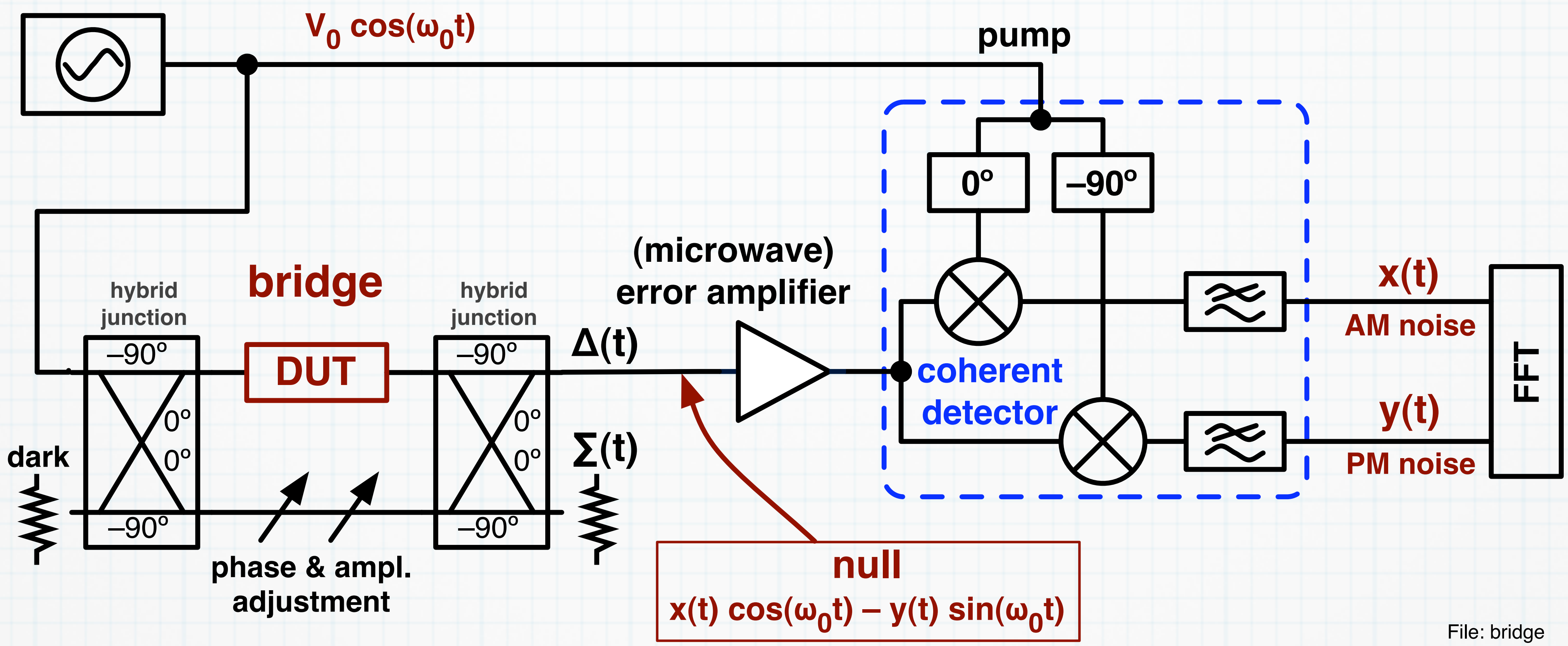
noise comparator



C. M. Allred, J. Res. NBS 66C no.4 p. 323-330, Oct-Dec 1962

C. M. Allred, A precision noise spectral density comparator, J. Res. NBS 66C no.4 p.323-330, Oct-Dec 1962

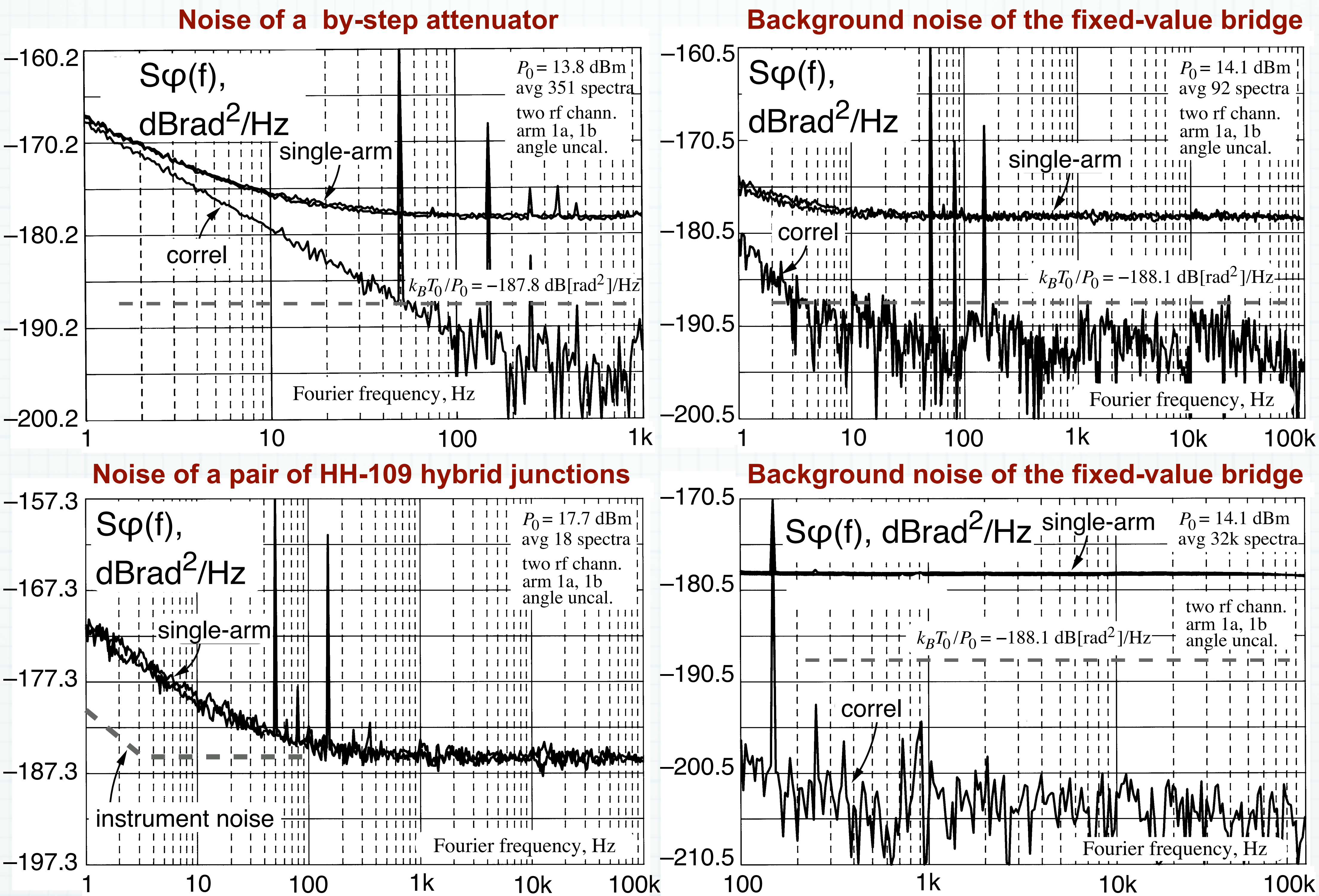
Bridge (Interferometric) Method



- Carrier suppression => the error amplifier cannot flicker: it does know ω_0
- High gain, due to the (microwave) error amplifier
- Low noise floor => the noise figure of the (microwave) error amplifier
- High immunity to the low-frequency magnetic fields due to the microwave amplification before detecting
- Rejection of the master oscillator’s noise
- Detection is a scalar product => signal-processing techniques

Derives from H. Sann, MTT 16(9) 1968, and F. Labaar, Microwaves 21(3) 1982
Later, E. Ivanov, MTT 46(10) oct 1998, and Rubiola, RSI 70(1) jan 1999

Example of results



Averaged spectra must be smooth

Average on m spectra: confidence of a point improves by $O(1/m^{1/2})$

interchange ensemble with frequency: smoothness $O(1/m^{1/2})$

A Final Word

- Review of PM noise and frequency (in)stability
- What happens in components
- Oscillators, how they work…
- Instruments,
  - Questions are still open
  - Correlation does not do what it promises
- Available here Thu morning and Fri all day
- Most of my stuff is on my web page
- Challenging questions are welcome at any time

enrico@rubiola.org

home page http://rubiola.org