

How to Calculate Bandwidth Smearing in Radio Synthesis Measurements

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1 The Answer

For a interferometer with a maximum baseline length B_{max} in meters with antenna elements with diameter D in meters the reduction in peak response to a point source at the edge of the primary beam is given by:

$$R_{\Delta\nu} = \frac{1}{\sqrt{1 + \beta_{max}^2}} \text{ (Gaussian Bandpass; Circular Gaussian Beam Taper)} \quad (1)$$

$$= \frac{\sqrt{\pi}}{2\sqrt{\ln 2}\beta_{max}} \text{erf} \left(\sqrt{\ln 2}\beta_{max} \right) \text{ (Square Bandpass; Circular Gaussian Beam Taper)} \quad (2)$$

$$\beta_{max} = \frac{\Delta\nu}{\nu} \frac{B_{max}}{2\sqrt{\ln 2}D} \quad (3)$$

2 Bandwidth Smearing Derivation

In the following I derive the relationship between the observing bandwidth, frequency, array maximum baseline length, array antenna element diameter and the amount of bandwidth smearing experienced by a point source in the interferometer field of view. A thorough treatment of the following can be found in Perley (1981, “The Effect of Bandwidth on the Synthesized Beam”, VLA Scientific Memorandum 138) and Bridle & Schwab (1989, “Synthesis Imaging in Radio Astronomy”, ASP Conference Series, Volume 6, Chapter 13). From Bridle & Schwab (1989; Equations 13–24 and 13–29), the reduction in the peak response from a point source in a given field of view of a radio interferometer is given by:

$$R_{\Delta\nu} = \frac{I}{I_0} = \frac{1}{\sqrt{1 + \beta_{max}^2}} \text{ (Gaussian Bandpass; Circular Gaussian Beam Taper)} \quad (4)$$

$$= \frac{\sqrt{\pi}}{2\sqrt{\ln 2}\beta_{max}} \text{erf} \left(\sqrt{\ln 2}\beta_{max} \right) \text{ (Square Bandpass; Circular Gaussian Beam Taper)} \quad (5)$$

where

$$\beta = \frac{\Delta\nu}{\nu} \frac{\theta_0}{\theta_{synth}} \quad (6)$$

where we have used the following definitions

- $I \equiv$ Peak intensity of the point source measured
- $I_0 \equiv$ Peak intensity of the point source with no bandwidth smearing
- $\Delta\nu \equiv$ Measurement bandwidth
- $\nu \equiv$ Observing frequency
- $\theta_0 \equiv$ Angular offset from the observing field center
- $\theta_{synth} \equiv$ Synthesized beam width
- erf is the mathematical *error function* defined by $\text{erf } x = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$

The main beam full-width at half-maximum (FWHM) beam width θ_{PB} defines the region on the sky over which we need to consider bandwidth smearing of a source. θ_{PB} is defined as follows (see Baars 2007, “The Paraboloidal Reflector Antenna in Radio Astronomy and Communication”, Chapter 4):

$$\theta_{PB} = \frac{(180 * 3600) b \lambda}{\pi D} \text{ arcsec}, \quad (7)$$

where b is the illumination taper factor. For a Gaussian beam $b = 1/\sqrt{\ln(2)} \simeq 1.2$. We can also approximate θ_{synth} in terms of the maximum baseline in the observation as follows:

$$\theta_{synth} \simeq \frac{(180 * 3600) \lambda}{\pi B_{max}} \text{ arcsec} \quad (8)$$

This allows us to write Equation 6 at its largest value in a given field of view (at the edge of θ_{PB} , or $\theta_0 = \frac{\theta_{PB}}{2}$) as follows:

$$\begin{aligned} \beta_{max} &= \frac{\Delta\nu}{2\nu\sqrt{\ln 2}} \frac{\lambda}{D} \frac{B_{max}}{\lambda} \\ &= \frac{\Delta\nu}{\nu} \frac{B_{max}}{2\sqrt{\ln 2} D} \end{aligned} \quad (9)$$

Figure 1, snatched from Bridle & Schwab (1989; Figure 13-1), shows $R_{\Delta\nu}$ as a function of β for a variety of bandpass shapes and synthesized beam tapers. Figure 2 shows the maximum bandwidth allowable for a 1% degradation in peak flux assuming a Gaussian bandpass.

To illustrate, assume the following:

- $\Delta\nu = 125 \text{ MHz}$

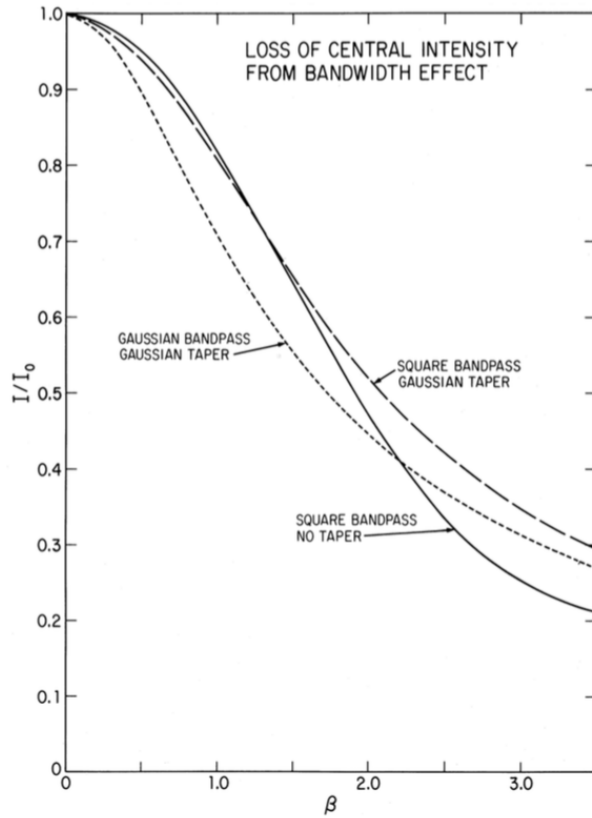


Figure 1: The reduction in peak response to a point source, I/I_0 , for each of the band shape and taper combinations discussed in Sections 2.2 through 2.4 in Bridle & Schwab (1989), plotted as a function of the dimensionless parameter β (extracted from Bridle & Schwab (1989)).

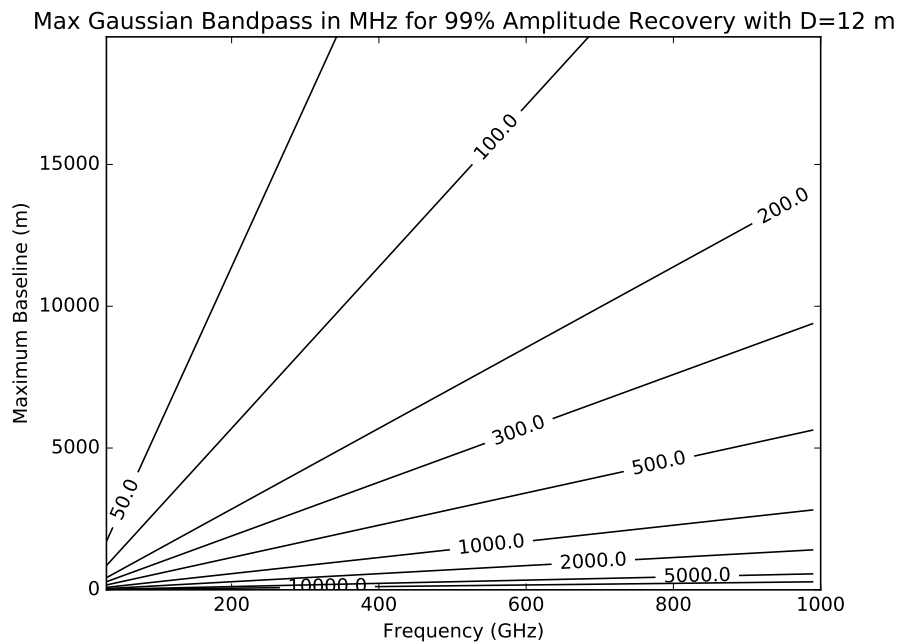


Figure 2: The maximum bandwidth allowed for a reduction in peak response to a point source over the field of view of 1% for a Gaussian bandpass.

- $\nu = 230$ GHz
- $B_{max} = 15$ km (the longest baseline for ALMA)
- $D = 12$ m (ALMA main array primary diameter)

in Equation 9, which results in $\beta_{max} \simeq 0.41$. Using this value of β_{max} in Equation 5 yields $R \simeq 0.93$ (Gaussian bandpass) and 0.96 (square bandpass), or a reduction of 7% and 3% in the peak intensity for a point source at the edge of the primary beam due to bandwidth smearing. Table 1 lists the maximum bandwidth ($\Delta\nu$) at all of the ALMA receiver band edge frequencies for Gaussian and square bandpass shape at two representative maximum baselines (B_{max}).

Table 1: ALMA Maximum Bandwidth To Avoid Amplitude Smearing^a

Frequency (GHz)	$\Delta\nu(Gaussian)$ (MHz)		$\Delta\nu(Square)$ (MHz)	
	$B_{max} = 500$ m	$B_{max} = 10,000$ m	$B_{max} = 500$ m	$B_{max} = 10,000$ m
35.0	199.30	9.97	195.92	9.80
50.0	284.72	14.24	279.88	13.99
67.0	381.52	19.08	375.04	18.75
90.0	512.49	25.62	503.79	25.19
84.0	478.33	23.92	470.20	23.51
116.0	660.55	33.03	649.33	32.47
125.0	711.80	35.59	699.71	34.99
163.0	928.18	46.41	912.42	45.62
211.0	1201.51	60.08	1181.10	59.06
275.0	1565.95	78.30	1539.35	76.97
373.0	2124.00	106.20	2087.92	104.40
385.0	2192.33	109.62	2155.09	107.75
500.0	2847.18	142.36	2798.82	139.94
602.0	3428.01	171.40	3369.78	168.49
720.0	4099.94	205.00	4030.30	201.52
787.0	4481.47	224.07	4405.35	220.27
950.0	5409.65	270.48	5317.76	265.89

^a Assuming $R_{\Delta\nu} = 0.99$, $D = 12$ m