How to Calculate Bandwidth Smearing in Radio Synthesis Measurements

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1 The Answer

For a interferometer with a maximum baseline length B_{max} in meters with antenna elements with diameter D in meters the reduction in peak response to a point source at the edge of the primary beam is given by:

$$\boxed{\begin{aligned} R_{\Delta\nu} &= \frac{1}{\sqrt{1+\beta^2_{max}}} \; (\textit{Gaussian Bandpass; Circular Gaussian Beam Taper}) && (1)\\ &= \frac{\sqrt{\pi}}{2\sqrt{\ln 2}\beta_{max}} \mathrm{erf}\left(\sqrt{\ln 2}\beta_{max}\right) \; (\textit{Square Bandpass; Circular Gaussian Beam Taper}) && (2)\\ \beta_{max} &= \frac{\Delta\nu}{\nu}\frac{B_{max}}{2\sqrt{\ln 2}D} && (3) \end{aligned}}$$

2 Bandwidth Smearing Derivation

In the following I derive the relationship between the observing bandwidth, frequency, array maximum baseline length, array antenna element diameter and the amount of bandwidth smearing experienced by a point source in the interferometer field of view. A thorough treatment of the following can be found in Perley (1981, "The Effect of Bandwidth on the Synthesized Beam", VLA Scientific Memorandum 138) and Bridle & Schwab (1989, "Synthesis Imaging in Radio Astronomy", ASP Conference Series, Volume 6, Chapter 13). From Bridle & Schwab (1989; Equations 13–24 and 13–29), the reduction in the peak response from a point source in a given field of view of a radio interferometer is given by:

$$\begin{aligned} R_{\Delta\nu} &= \frac{I}{I_0} \\ &= \frac{1}{\sqrt{1+\beta^2_{max}}} \; (\textit{Gaussian Bandpass; Circular Gaussian Beam Taper}) && (4)\\ &= \frac{\sqrt{\pi}}{2\sqrt{\ln 2}\beta_{max}} \mathrm{erf}\left(\sqrt{\ln 2}\beta_{max}\right) \; (\textit{Square Bandpass; Circular Gaussian Beam Taper}) && (5) \end{aligned}$$

where

$$\beta = \frac{\Delta\nu}{\nu}\frac{\theta_0}{\theta_{synth}} \tag{6}$$

where we have used the following definitions

- $I \equiv$ Peak intensity of the point source measured
- $I_0 \equiv$ Peak intensity of the point source with no bandwidth smearing
- $\Delta\nu \equiv$ Measurement bandwidth
- $\nu \equiv$ Observing frequency
- $\theta_0 \equiv$ Angular offset from the observing field center
- $\theta_{synth} \equiv$ Synthesized beam width
- erf is the mathematical *error function* defined by $\mathrm{erf}\, x = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}\,dt$

The main beam full-width at half-maximum (FWHM) beam width θ_{PB} defines the region on the sky over which we need to consider bandwidth smearing of a source. θ_{PB} is defined as follows (see Baars 2007, "The Paraboloidal Reflector Antenna in Radio Astronomy and Communication", Chapter 4):

$$\theta_{PB} = \frac{(180 * 3600)}{\pi}\frac{b\lambda}{D}\ \mathrm{arcsec}, \tag{7}$$

where b is the illumination taper factor. For a Gaussian beam $b = 1/\sqrt{\ln(2)} \simeq 1.2$. We can also approximate θ_{synth} in terms of the maximum baseline in the observation as follows:

$$\theta_{synth} \simeq \frac{(180 * 3600)\lambda}{\pi B_{max}}\ \mathrm{arcsec} \tag{8}$$

This allows us to write Equation 6 at its largest value in a given field of view (at the edge of θ_{PB}, or $\theta_0 = \frac{\theta_{PB}}{2}$) as follows:

$$\begin{aligned}\beta_{max} &= \frac{\Delta\nu}{2\nu\sqrt{\ln 2}}\frac{\lambda}{D}\frac{B_{max}}{\lambda} \\ &= \frac{\Delta\nu}{\nu}\frac{B_{max}}{2\sqrt{\ln 2}D}\end{aligned} \tag{9}$$

Figure 1, snatched from Bridle & Schwab (1989; Figure 13-1), shows $R_{\Delta\nu}$ as a function of β for a variety of bandpass shapes and synthesized beam tapers. Figure 2 shows the maximum bandwidth allowable for a 1% degradation in peak flux assuming a Gaussian bandpass.

To illustrate, assume the following:

- $\Delta\nu = 125\,\mathrm{MHz}$

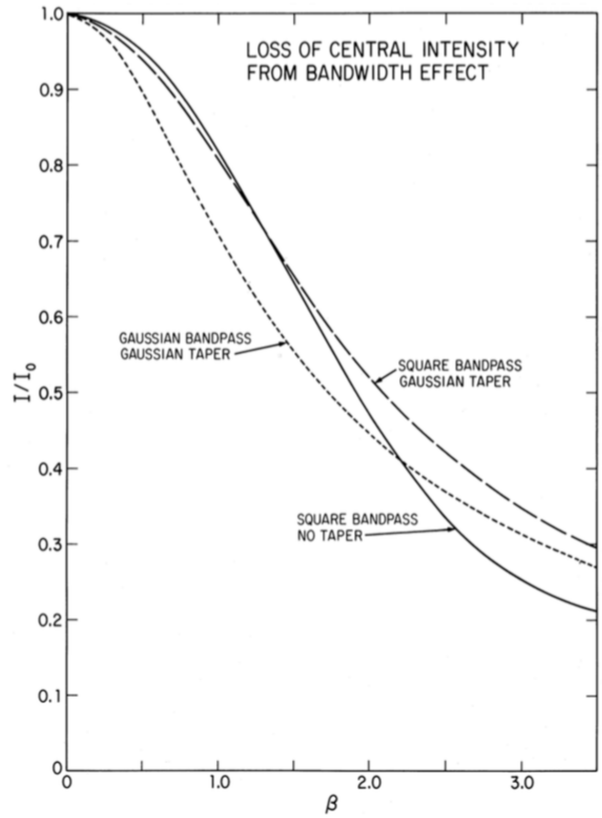

Figure 1: The reduction in peak response to a point source, $\frac{I}{I_0}$, for each of the band shape and taper combinations discussed in Sections 2.2 through 2.4 in Bridle & Schwab (1989), plotted as a function of the dimensionless parameter β (extracted from Bridle & Schwab (1989)).

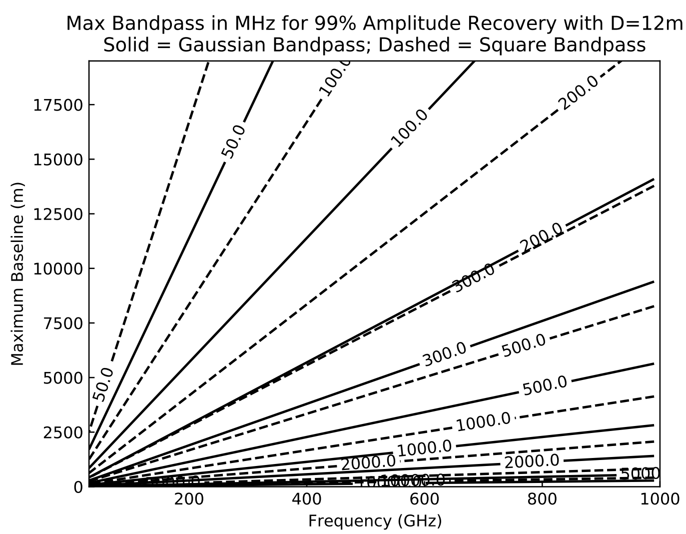

Figure 2: The maximum bandwidth allowed for a reduction in peak response to a point source over the field of view of 1% for a Gaussian (solid) and Square (dashed) bandpass.

- $\nu = 230\,\mathrm{GHz}$
- $B_{max} = 15\,\mathrm{km}$ (the longest baseline for ALMA)
- $D = 12\,\mathrm{m}$ (ALMA main array primary diameter)

in Equation 9, which results in $\beta_{max} \simeq 0.41$. Using this value of β_{max} in Equation 5 yields $R \simeq 0.93$ (Gaussian bandpass) and 0.96 (square bandpass), or a reduction of 7% and 3% in the peak intensity for a point source at the edge of the primary beam due to bandwidth smearing. Table 1 lists the maximum bandwidth ($\Delta\nu$) at all of the ALMA receiver band edge frequencies for Gaussian and square bandpass shape at two representative maximum baselines (B_{max}).

Table 1: ALMA Maximum Bandwidth To Avoid Amplitude Smearing[a,b]

Frequency (GHz)	$\Delta\nu(Gaussian)$ (MHz)		$\Delta\nu(Square)$ (MHz)	
	$B_{max} = 500\,\mathrm{m}$	$B_{max} = 10{,}000\,\mathrm{m}$	$B_{max} = 500\,\mathrm{m}$	$B_{max} = 10{,}000\,\mathrm{m}$
35.0	199.30	9.97	292.31	14.62
50.0	284.72	14.24	417.58	20.88
67.0	381.52	19.08	559.56	27.98
90.0	512.49	25.62	751.64	37.58
84.0	478.33	23.92	701.53	35.08
116.0	660.55	33.03	968.78	48.44
125.0	711.80	35.59	1043.95	52.20
163.0	928.18	46.41	1361.31	68.07
163.0	928.18	46.41	1361.31	68.07
211.0	1201.51	60.08	1762.18	88.11
211.0	1201.51	60.08	1762.18	88.11
275.0	1565.95	78.30	2296.68	114.83
275.0	1565.95	78.30	2296.68	114.83
373.0	2124.00	106.20	3115.14	155.76
385.0	2192.33	109.62	3215.36	160.77
500.0	2847.18	142.36	4175.79	208.79
602.0	3428.01	171.40	5027.65	251.38
720.0	4099.94	205.00	6013.14	300.66
787.0	4481.47	224.07	6572.69	328.63
950.0	5409.65	270.48	7934.00	396.70

[a] Assuming $R_{\Delta\nu} = 0.99$, $D = 12\,\mathrm{m}$

[b] Note that $\frac{\Delta\nu(Square)}{\Delta\nu(Gaussian)} = 1.46664$ at all frequencies for a given B_{max}