Antenna Control Systems: From PI to \( H_\infty \)

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Abstract

This paper discusses the compensation of antenna-pointing errors following the recent analysis and retrofit of the NASA Deep Space Network antenna-control systems. The desired high-frequency communications with spacecraft (at Ka-band) require improved pointing precision over lower-frequency communications (at X-band). The quality of the antenna drives (hardware), the control algorithm (software), and the physical structure of the antenna (in terms of thermal deformations, gravity distortions, encoder mounting, and wind gusts) all influence pointing precision, and create the challenging task of remaining within the required pointing-error budget.

Three control algorithms – PI (proportional-and-integral), LQG (linear-quadratic-Gaussian), and \( H_\infty \) – are discussed, and their basic properties, tracking precision, and limitations as applied to antenna tracking are addressed. The paper shows that the PI algorithm is simple and reliable, but its performance is limited. It also explains how significant improvements in tracking precision are achieved when implementing the LQG control algorithm or the \( H_\infty \) control algorithm. Still, pointing precision attributable to software modification is limited. It is pointed out that an additional increase of tracking precision requires concurrent improvements in the antenna drives.

Keywords: Control systems, control engineering, antennas, pointing systems, reflector antennas, reflector antenna mechanical factors, linear-quadratic-Gaussian control, H-infinity control

1. Introduction: Why Bother?

The NASA Deep Space Network (DSN) antennas, retrofitted for Ka-band (32 GHz) spacecraft communication, require a tracking precision of 2 arcsec. This is a challenging requirement for the antenna-control engineer, given the large physical dimensions (34 and 70 meter dishes: see Figure 1). Upgrades from simple proportional and integral (PI) controllers to linear-quadratic-Gaussian (LQG) controllers that track with higher precision were made, as reported in [1-3]. The \( H_\infty \) controllers, known to be the most advanced, are still being considered for implementation in the antenna servo system, as discussed in [4,5].

The purpose of this paper is to describe control algorithms with a potential for implementation: from the simplest to the most advanced, including their properties, the specification of pointing performance, and their limits. The paper is based predominantly on the author's analytical and hands-on experience with the DSN antennas. This work also serves as a short characterization of the antenna engineering, and will help clarify step towards pointing improvements.

Figure 1. The 34-meter DSS-15 antenna (foreground), and the 70-meter DSS-14 antenna (background), at the Goldstone Deep Space Communication Complex.
2. Antenna Under Test: The Control Engineer Point-of-View

The 34-meter Deep Space Network antenna has been investigated. Shown in Figure 1, this antenna is located at the Deep Space Communication Complex at Goldstone, California. DC electrical motors rotate the antenna with respect to the azimuth (vertical) axis and the elevation (horizontal) axis.

The antenna model (called also a rate-loop model) includes the structure and drives (motors, gearboxes, and amplifiers). The encoder reading, $y$ [deg], is the output to the rate-input signal $u$ [deg/s] (see Figure 2a). In this model, wind gusts, $w$, are additive disturbances to the control signal. This model is called a rate-loop model. It is separate for the azimuth and the elevation axes.

The block diagram of the closed-loop system consists of the antenna and controller interconnected with the feedback loop (see Figure 2b). The same system with the feedback loop open is called an open-loop system. The controller is the computer software that guides the antenna, depending on the actual position versus the commanded position. It consists of two inputs: the encoder position, $y$ [deg], and the commanded position (or simply, command), $r$ [deg], and an output rate, $u_c$ [deg/s] that drives the antenna. In order to save space, the differences between the azimuth and elevation axes will be considered negligible, and only the azimuth model will be considered. Antenna motions along the azimuth and elevation axes are not coupled; therefore, the system in Figures 2a and 2b represents either the azimuth or elevation control system.

The state-space model of the antenna was developed using the measured encoder response to white noise at a sampling rate of 40 Hz. A system-identification procedure was used to derive the state-space model between the recorded input and output data. The resulting transfer function consists of an integral part (or rigid-body part) that dominates the lower frequencies (below 1 Hz), and is characterized with $-20$ dB/deg slope of magnitude and with a phase of $-90$ deg. At higher frequencies – above 1 Hz – the transfer function shows flexible deformations, characterized by resonant
peaks in the magnitude and corresponding phase shifts of 180 deg. The magnitude and phase of the system’s transfer function (obtained from the antenna field tests) are shown in Figure 3. The validity of these models was checked with the closed-loop pointing simulations, and with closed-loop field-test data. These verified that the analysis results presented below are not purely academic.

3. Performance Criteria and Design Goals: Wishful Thinking?

In the following sections, controllers are analyzed and performance is evaluated in terms of the following characteristics:

- The settling time and overshoot of a step response (see Figure 4a). The settling time is defined as the time at which the antenna-encoder output arrives and remains within a 3% threshold of the nominal value of the step command. A 15 s settling time due to a unit-step command is illustrated in Figure 4a. The settling time indicates how fast the antenna reacts to the command, and the size of the bandwidth. Minimal settling times are desired. Overshoot (in percent) is the relative difference between the maximal encoder output and the commanded step. Figure 4a shows an 18% overshoot. The goal is to have overshoot below 20%.

- The amplitude and settling time of a disturbance step input. Wind disturbance, in the form of rapid (step-wise) signals, is suppressed by controller counteraction. Smaller amplitude and reaction time indicate a better controller. The goal is to minimize the amplitude.

- The bandwidth of the closed-loop transfer function (Figure 4b). The bandwidth is the frequency at which the magnitude drops 3 dB (to 70.7%) below the zero-dB level. Figure 4b illustrates a bandwidth of 0.14 Hz. A wider bandwidth implies faster and more precise control; therefore, the goal is to maximize bandwidth.

- The magnitude of the disturbance transfer function. A controller with a lower transfer-function magnitude will better reject wind disturbances. The goal is to minimize the magnitude.

- The steady-state error in rate offsets. The existence of steady-state rate error represents a lagging of the antenna when com-
The azimuth-axis responses of the three types of controllers to a 10 mdeg/s rate offset. 

Figure 8. The azimuth-axis responses of the three types of controllers to a 10 mdeg/s rate offset.

The azimuth-axis PI controller performance in response to a 10 mdeg step. 

Figure 9a. The azimuth-axis PI controller performance in response to a 10 mdeg step.

The azimuth-axis PI controller performance in response to a 10 mdeg/s rate disturbance demanded at a constant rate. Zero steady-state error is an achievable goal.

The phase and gain stability margin are measures of robustness. They indicate how much the system gain and phase can be perturbed before destabilizing the closed-loop system. Gain crossover is the frequency at which the open-loop (antenna-plus-controller) magnitude first reaches the value of unity. Thus, gain margin is the factor by which the open-loop magnitude must be multiplied to destabilize the system. The phase crossover is the frequency at which the open-loop phase angle first reaches the value of -180 deg, so phase margin is the number of degrees of delay that will destabilize the system. The margins of the antenna with a PI controller are shown in Figure 5.

The controller design goals listed above are highly interdependent. For example, a short settling time corresponds to a wide bandwidth, and a small amplitude of the disturbance step response corresponds to a small wind root-mean-square error: an improvement in one tends to cause improvement or degradation in the other. Each goal is assigned a weighting, in order to optimize performance according to specifications. Determining this weighting is not an obvious task, and weights are often selected based on the designer’s previous experience, by using a trial-and-error basis, or both.

References

IEEE Antennas and Propagation Magazine, Vol. 43, No. 1, February 2001

55
Table 1. The rms servo error in 10 m/s wind gusts.

<table>
<thead>
<tr>
<th>Controller</th>
<th>Servo Error (arcsec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PI (azimuth)</td>
<td>1.8°</td>
</tr>
<tr>
<td>PI (elevation)</td>
<td>5.8°</td>
</tr>
<tr>
<td>LQG (azimuth)</td>
<td>0.1°</td>
</tr>
<tr>
<td>LQG (elevation)</td>
<td>0.3°</td>
</tr>
<tr>
<td>(H_\infty) (azimuth)</td>
<td>0.08</td>
</tr>
<tr>
<td>(H_\infty) (elevation)</td>
<td>0.18</td>
</tr>
</tbody>
</table>

\(^{\dagger}\)From measurements [4].

Figure 10a. The stability margins of the PI controller.

Figure 10b. The stability margins of the LQG controller.

Table 2. Stability margins.

<table>
<thead>
<tr>
<th>Controller</th>
<th>Gain Margin (dB)</th>
<th>Phase Margin (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PI (azimuth)</td>
<td>10.3</td>
<td>69</td>
</tr>
<tr>
<td>PI (elevation)</td>
<td>6.2</td>
<td>69</td>
</tr>
<tr>
<td>LQG (azimuth)</td>
<td>7.1</td>
<td>45</td>
</tr>
<tr>
<td>LQG (elevation)</td>
<td>4.8</td>
<td>38</td>
</tr>
<tr>
<td>(H_\infty) (azimuth)</td>
<td>7.9</td>
<td></td>
</tr>
<tr>
<td>(H_\infty) (elevation)</td>
<td>5.1</td>
<td></td>
</tr>
</tbody>
</table>

4. PI Controller: A Convenient Way to Begin

PI controllers are no longer widely used to control large antennas, because they are unable to suppress antenna vibrations, and they are unable to meet the stringent pointing requirements. Still, subsystems such as the antenna subreflector can be controlled sufficiently with a PI controller, and an outline of this simplistic model (Figure 6) will help the reader to understand the properties of LQG and \(H_\infty\) controllers. In the controller, the command, \(r\), and the encoder signal, \(y\), are compared, and the result is the servo error, \(e = r - y\) (Figure 6). The servo error is integrated, obtaining the integral of the error, \(e_i\). The controller output, \(u_c\), is a combination of the servo error multiplied by the proportional gain, \(k_p\), and the integral of the error multiplied by the integral gain, \(k_i\).

In order to review the PI controller action, consider first a proportional controller (assuming a zero integral gain, \(k_i = 0\)), with a proportional gain \(k_p = 0.5\). The response of this closed-loop system to a 10 mdeg step command (shown in Figure 7a) has no overshoot and a 7 s settling time. On the other hand, the response to the 10 mdeg/s rate offset in Figure 7b has a constant servo error (or lagging) of 20 mdeg. It is desirable to have zero lag, which can
be achieved by increasing the proportional gain. Indeed, increasing the gain to 1.6 produces a smaller lag (6 mdeg), but the system is almost unstable (see the step response in Figure 7a). Thus, the proportional controller cannot eliminate the lagging. It is an integral gain that eliminates the lagging. A simulation of a proportional gain of $k_p = 0.5$ and an integral gain of $k_i = 0.1$ responded to the 10 mdeg/s rate offset as shown in Figure 8. Indeed, the rate-offset response had zero steady-state error, due to the action of the integrator in the controller (for non-zero steady-state error, the integral error would grow indefinitely, causing the controller to act accordingly).

On the other hand, the integrator of the PI controller produces an overshoot. In fact, the antenna response to a 10 mdeg step command, shown in Figure 9a, has 18% overshoot. It also has an excessive settling time of 15 s.

The response of the PI controller to a 10 mdeg/s disturbance step (pictured in Figure 9b) is slow and of large amplitude. The

The azimuth-axis LQG controller performance in response to a 10 mdeg/s rate disturbance.

Figure 12a. The azimuth-axis LQG controller performance in response to a 10 mdeg step.

Figure 12b. The azimuth-axis LQG controller performance in response to a 10 mdeg/s rate disturbance.

Figure 12c. The magnitude of the transfer function, from command to the encoder, for the azimuth-axis LQG controller.

Figure 12d. The magnitude of the disturbance transfer function, from disturbance to the encoder, for the azimuth-axis LQG controller.

Figure 11. A block diagram of the LQG and $H_\infty$ controllers.
servo error in 10 m/s wind gusts is quite large: 5.8 arcsec (see Table 1).

The magnitude of the transfer function of the PI system (from the command to the encoder), and the disturbance transfer function (from the wind disturbance to the encoder), are shown in Figures 9c and 9d, respectively. In this case, the system transfer function shows a narrow bandwidth of 0.1 Hz, and a strong resonance peak at 2 Hz. The disturbance transfer function tends to zero for low frequencies (translating into a total suppression of the steady-state disturbances), and has a strong resonance peak at 2 Hz. The magnitude of the disturbance transfer function is too high to produce good disturbance-rejection properties.

The large stability margins of the PI system are shown in Figure 10a: a gain margin of 10.3 dB and a phase margin of 69 deg (see Table 2) indicate that this servo system is robust.

The above analysis implies that the presented PI servo performance is inadequate. How can it be improved? It has already been noted that increased proportional gain improves the tracking properties (a faster step response, and increased bandwidth), and reduces the servo error in wind. Unfortunately, the gain increase is limited, since it leads to unstable structural vibrations. Therefore, the PI controller should be upgraded such that it controls structural vibrations.

5. LQG Controller: A Significant Improvement

It has been noted that the bandwidth, the speed of the system's response, and the disturbance-suppression abilities of the PI controller improve with the increase of the controller's proportional gain, up to a limiting value at which the antenna vibrates. If the vibrations could be sensed and controlled during the gain increase, the performance could be further improved. Currently, the only sensor available is the encoder. When one performs Fourier transformation of the encoder measurements, one notices that they include antenna vibrations. Thus, the antenna vibrations can be recovered from the encoder data. This can be done using an estimator, as in Figure 11. The estimator is an analytical antenna model, driven by the same input as the antenna itself (the rate

Figure 13a. The azimuth-axis \( H_\infty \) controller performance in response to a 10 mdeg step.

Figure 13b. The azimuth-axis \( H_\infty \) controller performance in response to a 10 mdeg/s rate disturbance.

Figure 13c. The magnitude of the transfer function, from command to the encoder, for the azimuth-axis \( H_\infty \) controller.

Figure 13d. The magnitude of the disturbance transfer function, from disturbance to the encoder, for the azimuth-axis \( H_\infty \) controller.
input, \( u_c \), and by the estimation error, \( e \) (the difference between the actual encoder reading, \( y_e \), and the estimated encoder reading, \( y_{est} \)). The error is amplified with the estimator gain, \( k_e \), to correct for transient dynamics (Figure 11). The estimator returns the antenna states, which consist of the estimated encoder reading (or noise-free encoder measurements), and the estimated states, \( x_f \), of the flexible deformations of the antenna structure. These states effectively supply the missing vibration measurements. The resulting controller’s output is a combination of the PI controller’s outputs and the flexible-mode controller’s output. The first takes care of the tracking motion; the latter suppresses the antenna vibrations (obtained as the estimated flexible states, amplified by the gain, \( k_f \)).

In the above configuration, the increased proportional and integral gains do not destabilize the closed-loop system, since the flexible-mode controller keeps the antenna vibrations suppressed. With this asset, one can expect an unlimited increase of the gains without destabilizing the closed-loop system. In fact, analytically, one can obtain outstanding performance. But such a system, although stable, is not robust: small variations of parameters can destabilize this nicely performing system. Therefore, the robustness, measured as gain and phase margins, is included in the evaluation of the LQG controller.

The performance of the LQG controller is illustrated with the response to a 10 mdeg step command (Figure 12a), a 10 mdeg/s disturbance step (Figure 12b), a 10 mdeg/s rate-offset (Figure 12b), and transfer functions: from command to encoder (Figure 12c), and from disturbance to encoder (Figure 12d). The step response has a small settling time of 2 s, and the disturbance step response has a low magnitude of short duration (2 s). The rate offset shows zero lagging (see Figure 8). The command transfer function has a wide bandwidth of 2 Hz, and the disturbance transfer function has a small magnitude (below 1). The stability margins are shown in Figure 10b. The gain margin is 7.1 dB, and the phase margin is 45 deg (see Table 2). These margins are large enough to consider this particular LQG controller robust (“better” controllers, with shorter response times, and disturbance rejection properties had small margins). The servo error in 10 m/s wind gusts is small: 0.08 arcsec (see Table 1). The stability margins are shown in Figure 10c. The gain margin is 7.9 dB, and the phase margin is 42 deg, as shown in Table 2. These are similar to the values for the LQG controller, and show controller robustness.

### 6. H\(_{\infty}\) Controller: We Did Our Best (or Did We?)

H\(_{\infty}\) controllers outperform LQG controllers in many applications. The structure of an H\(_{\infty}\) controller is similar to that of the LQG controller, but its parameters are obtained from a different algorithm. While the LQG controller minimizes the system H\(_2\) norm (its rms response to the white-noise input), the H\(_{\infty}\)-controller algorithm minimizes the system’s H\(_{\infty}\) norm (in the case of a single-input-single-output system, the system H\(_{\infty}\) norm is the maximum magnitude of its transfer function).

The H\(_{\infty}\) controller was designed by shaping the disturbance input. A filter of the Davenport wind-spectrum profile was used as a shaping (or weighting) factor. More on wind spectra can be found in [8] and [2]. The performance of the antenna was evaluated with the servo error and its integral. Separate controllers were designed for the azimuth and elevation axes, and the results are summarized in Table 1. The table shows very small servo errors in 10 km/h wind gusts.

Additional results for the azimuth axis is shown in Figure 13. The figures show a very small settling time (1.2 s), a small overshoot (less than 10%), and a wide bandwidth (over 2 Hz). These features significantly exceed the LQG controller performance. The response to the 10 mdeg/s rate offset is shown in Figure 8. The response has zero steady-state error and a short settling time (below 1 s). The disturbance transfer function (in Figure 13d) has a very low magnitude (below 0.1). The servo error in 10 m/s wind gusts is small: 0.08 arcsec (see Table 1). The stability margins are shown in Figure 10c. The gain margin is 7.9 dB, and the phase margin is 42 deg, as shown in Table 2. These are similar to the values for the LQG controller, and show controller robustness.

### 7. Hardware Restrictions: An Obstacle to Improvement

In spite of these outstanding results, simulations and measurements indicate that the H\(_{\infty}\) controller cannot be implemented "as is," because the existing antenna drives (the motors and gearboxes) operate under acceleration limits that would effectively cancel out the benefits of an H\(_{\infty}\) controller. These limits (on the input signal, \( u \), that drives the antenna) are applied to prevent overloading of the motors. For the 34-meter antenna, the acceleration limits are \( \pm 0.4 \, \text{deg/s}^2 \). Notice, however, that the above H\(_{\infty}\) controller, in response to a 10-mdeg step offset, accelerates up to 40 \( \text{deg/s}^2 \), as shown in Figure 14. The acceleration limiter cuts off the signal, which would have peaked at 40 \( \text{deg/s}^2 \), at 0.4 \( \text{deg/s}^2 \). This produces non-linear dynamics, which destabilize the system (the estimator is linear, and cannot reproduce the acceleration limits). This issue of limiters applies not only to the H\(_{\infty}\) controllers, but also to "over-performing" LQG controllers. The LQG controller presented in Section 5 was selected such that the limits were not violated.

One way to avoid excessive accelerations is to implement a command pre-processor in the software, which modifies the antenna commands such that they never reach or exceed the rate and acceleration limits, as in [7]. The pre-processor will command the antenna with an unmodified command if the limits are not exceeded, and with the maximum rate and/or accelerations if the limits are exceeded. This technique has been proven effective when implemented with LQG and H\(_{\infty}\) controllers in calm (low-wind)
conditions. However, the presence of wind gusts generates additional antenna dynamics that are fed back to the controller. The controller, in turn, produces the rate command, \( U \), to fight the dynamics. This rate command exceeds the limit, introduces non-linear behavior, and destabilizes the system. This phenomenon indicates that improvement of the LQG and \( H_\infty \) controllers’ performance remains in a direct relationship to the magnitude of the acceleration limits. Replacing the existing motors and gearboxes with more-powerful units would allow for relaxation of the acceleration limits. Thus, there is a point at which further improvement in antenna performance will require not only modifications of the control software (the algorithm), but also of the hardware, as well (more powerful motors, stronger gearboxes, and strengthening of the antenna structure).

8. Conclusions

It has been shown that the LQG controller significantly outperformed the PI controller in terms of reaction time, bandwidth, pointing precision, and pointing error in wind gusts. Fortunately, the upgrade from the PI to LQG controller requires modifications of the antenna software only. The implementation of an \( H_\infty \) controller takes the pointing improvement one step further, but is restricted by the hardware limitations, namely the available motor torque. Thus, the upgrade from LQG to \( H_\infty \) controllers (or to more “aggressive” LQG controllers) requires not only implementation of new algorithms, but also replacement of motors and gearboxes with more-powerful units, and possibly some structural modifications. This conclusion, although not be totally unexpected, is not easily accepted. It requires costly modifications of the antenna hardware, and it should rather be addressed during the design process of a newly built antenna.

9. Acknowledgement

The research described in this paper was carried out by the Jet Propulsion Laboratory, California Institute of Technology, under a contract with the National Aeronautics and Space Administration.

10. References


Introducing the Feature Article Author

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