System temperature calculation and data weighting for the
Zpectrometer
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\textbf{Abstract}

This note gives the conversion from the Zpectrometer’s total power monitor point to system
temperature and shows how the result can be used for data weighting. At Ka-band, where
the Zpectrometer operates, the atmospheric transmission toward the source varies with
changing weather and source elevation. A system temperature, even if somewhat inexact
in absolute value, is very useful for optimally weighting individual spectra that will be
combined to assemble a long integration.

1 \textbf{System temperature measurement}

The Zpectrometer system contains a total power detector that monitors power after one of
the first amplifiers in the Zpectrometer’s internal channelizing downconverter. Filtering before
the detector selects much of the Ka-band where the receiver has good noise performance. The
choice of frequencies, 27.8–36.6 GHz on the sky, also included fabrication considerations for the
filter: covering 7.2–16 GHz in the IF, this filter has more than an octave bandwidth.

A simple model for the detector’s output has a fixed term and one that depends on sky
transmission and elevation:

\begin{equation}
V_{\text{det}} = K \left[ T_{\text{rec}} + (1 - t_z^{AM}) T_{\text{sky}} \right],
\end{equation}

where \( K \) is a constant with units volts per kelvin, \( T_{\text{rec}} \) is the receiver temperature, \( t_z \) is the
zenith atmospheric transmission, \( AM = 1/\cos(z) \) is the airmass, the secant of the zenith angle \( z \)
toward the source, and \( T_{\text{sky}} \) is the opacity-weighted physical temperature of the sky. Extending
the model by incorporating terms with little or no elevation dependence into an effective receiver
temperature may be useful. While not exact, adding a few kelvins to account for spillover is a
sensible practical approximation at high elevations.

The system temperature is the noise scaled to the top of the atmosphere, or

\begin{equation}
T_{\text{sys}} = \frac{1}{t_z^{AM}} \left[ T_{\text{rec}} + (1 - t_z^{AM}) T_{\text{sky}} \right].
\end{equation}

Solving equation (1) for the atmospheric transmission,

\begin{equation}
t_z^{AM} = \frac{T_{\text{sky}} + T_{\text{rec}} - V_{\text{det}}/K}{T_{\text{sky}}},
\end{equation}

and inserting the result into equation (2) gives the system temperature,

\begin{equation}
T_{\text{sys}} = \frac{T_{\text{sky}}}{T_{\text{sky}} + T_{\text{rec}} - V_{\text{det}}/K} \frac{V_{\text{det}}}{K}.
\end{equation}
Measurement with hot and cold thermal loads of known temperatures \( T_H \) and \( T_C \) at the receiver input gives \( K \) and \( T_{rec} \):

\[
K = \frac{v_{detH}}{T_H}
\]

\( (5) \)

(data from the cold load or the difference also work, but then the deviation is smaller) and

\[
T_{rec} = \frac{T_H - YT_C}{Y - 1},
\]

\( (6) \)

the usual Y-factor method for measuring receiver temperature with

\[
Y = \frac{v_{detH}}{v_{detC}}.
\]

\( (7) \)

It is important to compensate for the detector’s zero-point offset in all calculations using absolute detector voltages (eqs. (5) and (7)). For a simple diode detector the offset has some temperature dependence; measurements of the Zpectrometer’s detector offset are always between \(-0.209 \text{ and } -0.216 \text{ V}\) compared with typical signal levels of about a volt. The offset has proven to be stable, presumably because the GBT receiver room is stable in temperature.

Receiver gain changes will change the scaling factor \( K \). We make relative system gain measurements by injecting an amplitude-modulated noise diode signal between one input horn and the first hybrid, then synchronously detecting the detector diode output. This modulated signal amplitude remained constant at the few percent level through much of March 2008. This implies similar long-term gain stability for the receiver, assuming that the noise diode power is constant.

The main uncertainty in establishing a system temperature is in choosing the opacity-weighted physical temperature of the atmosphere, \( T_{sky} \). In principle, observations at very high airmass give \( T_{sky} \), but low opacity at Ka-band and ground pickup reduce the measurement’s accuracy. Assuming a ground-level ambient temperature will be an overestimate unless the atmosphere is extremely optically thick. Taking zero Celsius is a possibility if water vapor is the dominant absorber, since water vapor turns to ice and loses emissivity at lower temperatures. If the main opacity is not from water vapor, then lower temperatures are suitable, with models suggesting approximately 230–250 K. Fortunately, the same properties that make \( T_{sky} \) difficult to measure make knowledge of its exact value relatively unimportant for Ka-band observations in good weather. Consider the first term in equation (4). This is close to unity when \( T_{sky} \gg T_{rec} - v_{det}/K \), the situation with the GBT’s low receiver temperature and when the sky’s emission temperature is low \((v_{det}/K \text{ small})\). In this case the bulk of the change with sky emission arises from the second term, \( v_{det}/K \), with only little modification from the first term.

2 Data weighting

An optimal data combination scheme accounts for changes in system temperature during the overall integration time. For a variety of reasons a very long observation is generally broken into shorter integrations, each of a few minutes, that are stored as separate files for subsequent averaging in a data analysis program. Individual integration times are then short compared with the changes in weather and source elevation that are the most common sources of system temperature differences. Assigning higher weight to data with good atmospheric transmission toward source is especially important at high frequencies, where the atmosphere is variable on timescales of hours or less.
An appropriate weighting factor in a mean value calculation is the reciprocal of the variance: the weight $w_i$ for the $i$th individual observation is proportional to $1/\sigma_i^2$. Using the empirical measured variance in each short integration is not justifiable from a statistical point of view. Thermal noise in the radiometric system produces a limit to the variance, however, and is an independent predictor of statistical noise. The radiometer equation gives the amplitude of post-detection fluctuations,

$$
\sigma = \frac{T_{\text{rec}}}{\sqrt{B\tau}},
$$

where $B$ is the pre-detection bandwidth and $\tau$ is the post-detection integration time. To account for atmospheric transmission, move to a reference plane above the atmosphere where this noise is

$$
\sigma^* = \frac{1}{t_{\text{AM}}} \frac{T_{\text{rec}}}{\sqrt{B\tau}} = \frac{T_{\text{sys}}}{\sqrt{B\tau}}.
$$

Other noise contributions, such as spillover or antenna ohmic loss, may add to $T_{\text{sys}}$.

With constant bandwidth for all short observations that make up the total integration on source, the average flux $\overline{S}_\nu$ from the $N$ individual flux measurements $S_{\nu,i}$ is

$$
\overline{S}_\nu = \frac{\sum_{i=1}^{N} \tau_i T_{\text{sys},i}^{-2} S_{\nu,i}}{\sum_{i=1}^{N} \tau_i T_{\text{sys},i}^{-2}} = \frac{1}{N} \sum_{i=1}^{N} w_i S_{\nu,i}.
$$

Equation (10) defines the normalized weight,

$$
w_i = \frac{\tau_i T_{\text{sys},i}^{-2}}{N^{-1} \sum_{i=1}^{N} \tau_i T_{\text{sys},i}^{-2}}.
$$

Equation (11) shows that the weighting is linearly proportional to time and to the square of $T_{\text{sys}}$. The square dependence means that weighting by $T_{\text{sys}}$ can easily be more important than weighting by time. This emphasizes the need for a good estimate of the system temperature.

Figure 1: Data weighting factor $w$ versus time taken during an approximately 12 hour period of Ka-band observing with the Zpectrometer in March 2008. Gaps in the plot are from pointing and other activities that switched the IF power from the Zpectrometer.
Figure 1 is a plot of the weighting factor versus time taken during an approximately 12 hour period of Ka-band observing with the Zpectrometer in March 2008. The figure shows somewhat unstable weather and a rising source at the beginning of the run, a long quiescent period near source transit, and the effect of decreasing transmission and elevation past 11:30 UT. The jump at the end of the trace shows the telescope’s return to the access position near zenith. Data in the figure come from the Zpectrometer’s total power detector using eq. (4) with $K = 0.0282 \text{ V/K}$ (35.5 K/V) and a receiver temperature of 18.6 K derived from an ambient–liquid nitrogen load measurement. The detector voltage was $v_{\text{det}} \approx 1 \text{ V}$ for these observations. Taking $T_{\text{sky}} = 270 \text{ K}$, the system temperature ranged from about 38 to 45 K. Although the fractional change seems small, its square dependence in the weighting function produces a significant departure from unity weights at lower airmasses. A modest amount of numerical experimentation confirms that the weighting factor is insensitive to an exact choice of $T_{\text{sky}}$ for $T_{\text{sky}} > 250 \text{ K}$.

Figure 1 shows the importance of weighting by $T_{\text{sys}}$ when combining data, even during a single day with good weather. Weighting is even more important for combining data from different days with different weather conditions.

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