

ALMA Holography Feeds: Reference and Transmitter

Bill Shillue
NRAO, Tucson
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Introduction

This document calculates beam patterns and other parameters for the ALMA reference and transmitter feeds

Frequency and Wavelength

$$\nu := 104 \cdot \text{GHz}$$

The analysis can be repeated for 78 GHz

$$\lambda := \frac{c}{\nu}$$

$$\lambda = 2.883 \text{ mm}$$

Requirements

Transmitter Beamwidth: 3-dB BW 4.6 degrees minimum at 104 GHz

Reference Beamwidth: 3-dB BW 4.6 degrees minimum at 104 GHz

Design is based on preliminary design by L. D'Addario dated 2001-01-06 which has a 10.9 degree half angle feed with a feed aperture of 45.76 mm and a phase correcting lens attached to the end of the feed. This gives the desired 4.6 degree beamwidth. We have modified the preliminary design, however, so that the beamwidth is about 4.8 degrees.

Feed Lens Analysis (after James Lamb mathcad sheet)

Horn Parameters

Half angle of feed horn: $\theta_h := 10.9 \cdot \text{deg}$

Aperture radius: $a_h := 21.5 \cdot \text{mm}$

Length: $l := \frac{a_h}{\tan(\theta_h)} \quad l = 111.648 \text{ mm}$

Slant length: $R := \frac{a_h}{\sin(\theta_h)} \quad R = 113.699 \text{ mm}$

Lens Parameters

Refractive index: $n := 1.464$

Lens flange thickness: $t := 5 \cdot \text{mm}$

Slant Length with Lens: $R_L := R + \frac{t}{\cos(\theta_h)}$ $R_L = 118.791 \text{ mm}$

Equation of Outer Surface of Lens. R_L is one point on this curve, allowing us to solve for focal length - f

$$R_{\text{LENS}} = \frac{f \cdot (n - 1)}{n - \cos(\theta)}$$

Focal length: $f := \frac{n - \cos(\theta_h)}{n - 1} \cdot R_L$ $f = 123.410 \text{ mm}$

Eccentricity: $e := \frac{1}{n}$ $e = 0.683$

$a := \frac{f}{1 + e}$ $a = 73.325 \text{ mm}$

$b := a \cdot \sqrt{1 - e^2}$ $b = 53.554 \text{ mm}$

Axial position as a function of radial coordinate:

$z(r) := \frac{a}{b} \cdot \sqrt{b^2 - r^2} + \sqrt{a^2 - b^2}$ Axial position is relative to

Angle relative to vertex as a function of radial coordinate:

$\theta_l(r) := \text{atan}\left(\frac{r}{z(r)}\right)$

Calculate limit of illumination pattern at lens:

$r := a_h$

$r = 21.500 \text{ mm}$

$r_{\text{max}} := \text{root}(\theta_l(r) - \theta_h, r)$

$r_{\text{max}} = 22.463 \text{ mm}$

$z(r_{\text{max}}) = 116.648 \text{ mm}$

Aperture Amplitude Pattern

Assuming that the horn supports only the balanced hybrid HE11 mode, the horn aperture pattern with no lens is given by

$$Af(\theta) := J_0\left(j_{01} \cdot \frac{\theta}{\theta_h}\right) \quad P := 2 \cdot \pi \cdot \int_0^{\theta_h} Af(u)^2 \cdot \sin(u) \, du \quad P = 0.031$$

$$\mathcal{A}f(\theta) := \frac{1}{\sqrt{P}} \cdot J_0\left(j_{01} \cdot \frac{\theta}{\theta_h}\right)$$

where j_{01} is the first root of J_0 . Transform aperture illumination pattern by meniscus lens of focal length f and refractive index n to get amplitude illumination pattern with lens.

$$B := \frac{1}{\left(a + \sqrt{a^2 - b^2}\right)} \quad \frac{1}{B} = 123.410 \text{ mm}$$

From Silver, Rad Lab Series #12, Sec 11-3:

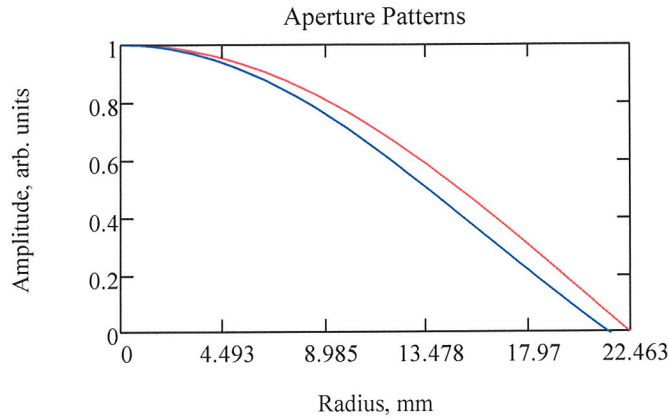
$$Al(r) := \sqrt{\frac{n - \cos(\theta_l(r))}{n \cdot (\cos(\theta_l(r)) - 1)}} \cdot \left(\text{if} \left(r = 0 \cdot \text{mm}, B, \frac{\sin(\theta_l(r))}{r} \right) \right) \cdot Af(\theta_l(r))$$

Normalisation factor

$$P_0 := 2 \cdot \pi \cdot \int_{0 \cdot m}^{r_{\max}} (|Al(r)|)^2 \cdot r \, dr \quad P_0 = 1.000$$

Now calculate aperture field amplitude.

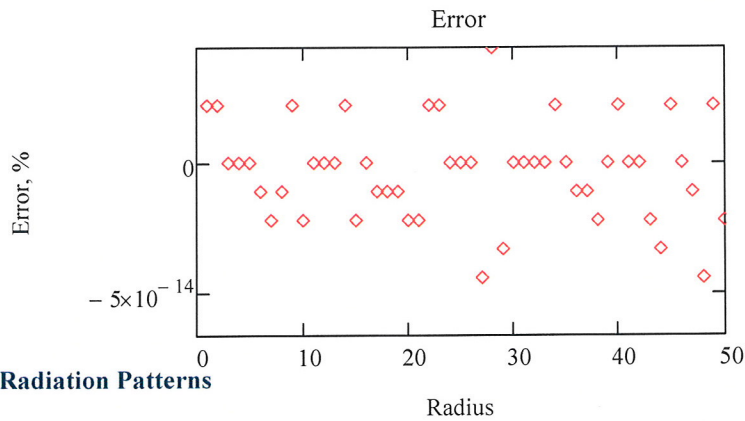
$$\begin{aligned} N &:= 50 & i &:= 0..N & r_{\mathcal{A}} &:= \frac{r_{\max}}{N} \cdot i \\ AL_i &:= Al(r_i) & AL_0 &:= AL_0 & \mathcal{A}_i &:= \frac{AL_i}{AL_0} & AF_i &:= Af\left(\text{atan2}(1, r_i)\right) & AF_0 &:= AF_0 & AFF_i &:= \frac{AF_i}{AF_0} \end{aligned}$$



Check of accuracy--the following condition from energy conservation should hold:

$$\int_0^{\theta} Af(u)^2 \cdot \sin(u) du = \int_0^r Al(\rho)^2 \cdot \rho d\rho \quad \theta_i := \theta(r_i)$$

$$P_{\text{horn}_i} := 2 \cdot \pi \cdot \int_0^{\theta_i} Af(u)^2 \cdot \sin(u) du \quad P_{\text{lens}_i} := 2 \cdot \pi \cdot \int_0^{r_{\text{max}}} (Al(\rho \cdot r_{\text{max}}))^2 \cdot \rho \cdot r_{\text{max}}^2 d\rho$$



Far-Field Radiation Patterns

Use Hankel transforms to obtain far-field patterns. To calculate the pattern of the horn only, take the aperture field amplitude defined above (truncated Bessel function), multiply by a spherical phase term, and transform to get

$$Gh(\theta) := \left(\int_{0.0}^{a_h} Af\left(\text{atan}\left(\frac{r}{l}\right)\right) \cdot \exp\left(-i \cdot \frac{\pi \cdot r^2}{R \cdot \lambda}\right) \cdot J_0\left(\frac{2 \cdot \pi}{\lambda} \cdot r \cdot \sin(\theta)\right) \cdot r dr \right)^2$$

By using the aperture field including the effect of the lens on the amplitude as calculated above the far-field amplitude pattern becomes

$$Gf(\theta) := \left(\int_{0.0}^{r_{\max}} Al(r) \cdot J_0\left(\frac{2 \cdot \pi}{\lambda} \cdot r \cdot \sin(\theta)\right) \cdot r \, dr \right)^2$$

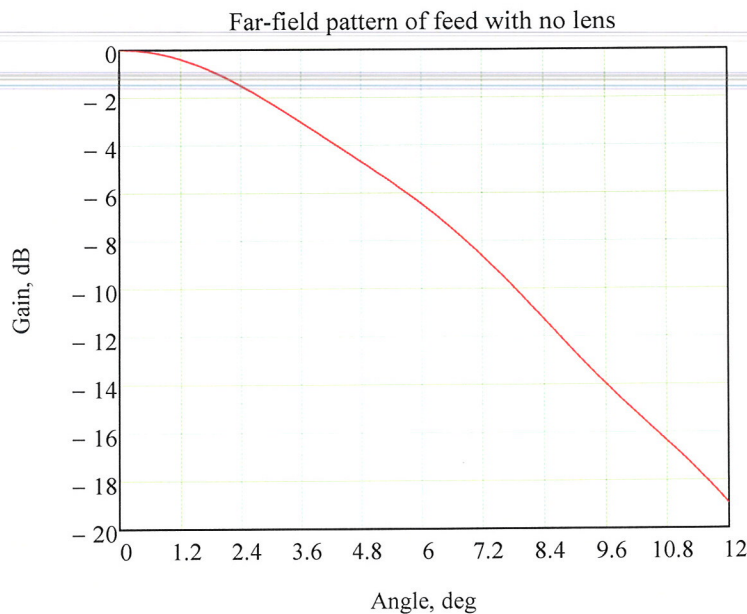
Now evaluate the above expressions numerically. For the corrugated horn with no lens the pattern is calculated over angles $0 < \theta < 2\theta_h$ where θ_h is the half angle of the horn.

$$N := 200 \quad i := 0..N \quad \theta_i := \frac{2 \cdot \theta_h}{N} \cdot i \quad \theta_N = 21.800 \text{ deg}$$

$$Gh := \overrightarrow{Gh(\theta)} \quad Gh_0 := Gh_0$$

Normalise to gain on boresight and convert to dB

$$Gh_{dB} := 10 \cdot \log\left(\frac{Gh}{Gh_0}\right)$$



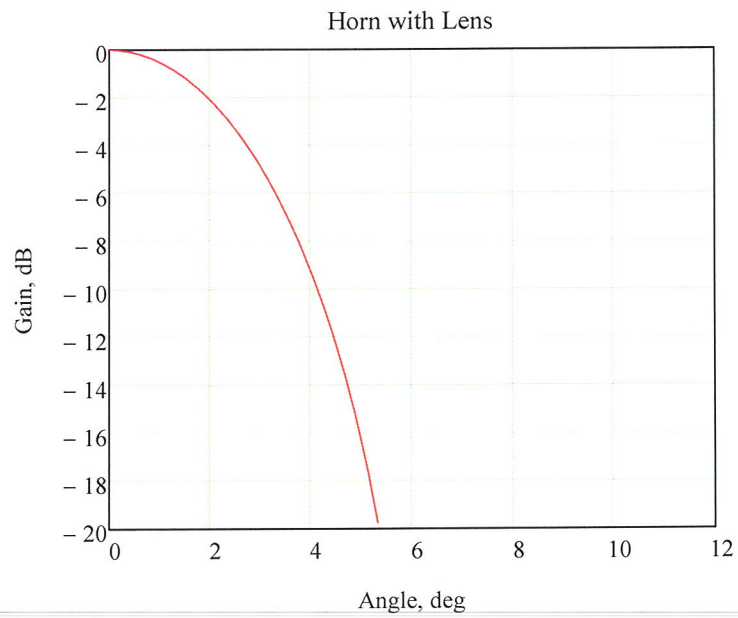
For the corrugated horn with the lens the pattern is calculated over angles $0 < \theta < 5\theta_f$ where θ_f is

$$\theta_f := \frac{\lambda}{2 \cdot r_{\max}} \quad \theta_i := \frac{5 \cdot \theta_f}{N} \cdot i \quad \theta_N = 18.382 \text{ deg}$$

$$Gf := \overrightarrow{Gf(\theta)} \quad Gf_0 := Gf_0 \quad Gf_{dB} := 10 \cdot \log\left(\frac{Gf}{\text{mm}^2}\right)$$

Normalise to gain on boresight and convert to dB

$$Gf_{dB} := 10 \cdot \log\left(\frac{Gf}{Gf_0}\right)$$



Lens Design and Parameters

The lens will be a two-surface lens with the inner surface an equiphase spherical surface and the outer surface an ellipsoid with focal point at the feed phase center

Refractive index: $n := 1.464$

Lens flange thickness: $t := 5 \cdot \text{mm}$

Slant Length with Lens: $R_L := R + \frac{t}{\cos(\theta_h)}$ $R_L = 118.791 \text{ mm}$

Equation of Outer Surface of Lens. R_L is one point on this curve, allowing us to solve for focal length - f

$$R_{\text{LENS}} = \frac{f \cdot (n - 1)}{n - \cos(\theta)}$$

Focal length: $f := \frac{n - \cos(\theta_h)}{n - 1} \cdot R_L$ $f = 123.410 \text{ mm}$

Generate a table of lens coordinates: $n := 10$ $r_{\text{max}} = 22.463 \text{ mm}$

$k := 0, 1 \dots 10$

$\text{rad}_k := r_{\text{max}} \cdot \frac{k}{n}$ $zz_k := z(\text{rad}_k)$ $\theta\theta_k := \text{atan}\left(\frac{\text{rad}_k}{zz_k}\right)$

	0
0	0.000
1	88.436
2	176.873
3	265.309
4	353.746
5	442.182
6	530.618
7	619.055
8	707.491
9	795.928
10	884.364

$\frac{\text{rad}}{25.4 \cdot \text{mm}} =$

$m - 1.0 \left(\frac{zz - 111.622 \cdot \text{mm}}{25.4 \cdot \text{mm}} \right) =$

	0
0	0.464
1	0.462
2	0.454
3	0.441
4	0.423
5	0.400
6	0.371
7	0.337
8	0.297
9	0.250
10	0.198

	0
0	0.000
1	1.043
2	2.089
3	3.140
4	4.199
5	5.269
6	6.353
7	7.454
8	8.576
9	9.723
10	10.900

$\theta\theta =$

de

Quarter wave matching of lens

The lens requires a quarter wave matching section. This will be done by machining circular grooves into the front and rear face of the lens. Groove parameters are shown below:

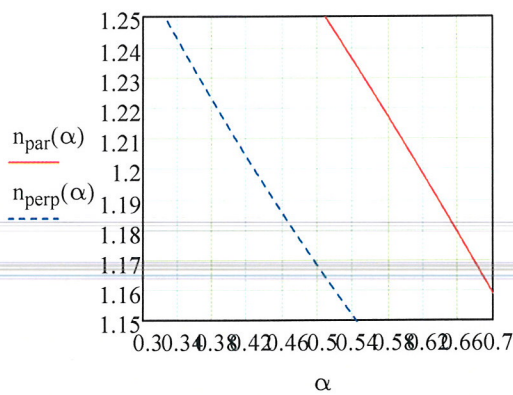
$$n_{\text{par}}(\alpha) := \sqrt{\alpha + (1 - \alpha) \cdot n^2}$$

$$n_{\text{perp}}(\alpha) := \sqrt{\frac{n^2}{1 - \alpha + \alpha \cdot n^2}}$$

$$n := 1.464$$

We want the effective dielectric constant to be \sqrt{n} and effective depth to be a quarter wavelength.

A full discussion of parameters is given in NRAO EDTN #170, "Dielectric Constants and Matching Groove Parameters for Millimeter Wavelengths", James W. Lamb



$$\text{groove width to pitch ratio: } \alpha := 0.5$$

$$n_{\text{par}}(0.5) = 1.254$$

$$n_{\text{perp}}(0.5) = 1.168$$

$$n_{\text{eff}} := \frac{n_{\text{par}}(0.5) + n_{\text{perp}}(0.5)}{2}$$

$$n_{\text{eff}} = 1.211$$

$$\lambda_{\text{ctr}} := \sqrt{\frac{c}{104 \cdot \text{GHz}} \cdot \frac{c}{78 \cdot \text{GHz}}}$$

$$\lambda_{\text{ctr}} = 3.329 \text{ mm}$$

$$\text{depth of grooves: } d := \frac{\lambda_{\text{ctr}}}{4 \cdot n_{\text{eff}}} \quad d = 0.687 \text{ mm}$$

$$\frac{c}{\lambda_{\text{ctr}}} = 90.067 \text{ GHz}$$

Groove pitch should be less than a half-wavelength, for convenience if we make the pitch equal to the depth, then the pitch is less than a quarter wave at the highest frequency (104 GHz), and about one third of a wavelength in the dielectric, so this should be OK

$$p := .687 \cdot \text{mm}$$

Wavelength in air at 104 GHz:

$$\lambda_{104_air} := 2.88 \cdot \text{mm} \quad \frac{p}{\lambda_{104_air}} = 0.239$$

Wavelength in lens at 104 GHz

$$\lambda_{104_lens} := \frac{\lambda_{104_air}}{n} \quad \lambda_{104_lens} = 1.967 \text{ mm} \quad \frac{p}{\lambda_{104_lens}} = 0.349$$

Wavelength in grooves at 104 GHz

$$\lambda_{104_grooves} := \frac{\lambda_{104_air}}{n_{\text{eff}}} \quad \lambda_{104_grooves} = 2.379 \text{ mm} \quad \frac{p}{\lambda_{104_grooves}} = 0.289$$

Throat Design

Throat design of a corrugated feed horn is a little bit mysterious. In general, one needs to choose a circular waveguide diameter to start with, and then choose slot depths for the first several slots. The slot depths start at $\lambda/2$ at the highest frequency and decrease to $\lambda/4$ at the mid-band frequency.

$$f_1 := 78 \cdot \text{GHz}$$

$$f_2 := 104 \cdot \text{GHz}$$

$$f_c := \sqrt{f_1 \cdot f_2}$$

$$f_c = 90.067 \text{ GHz}$$

$$\lambda_1 := \frac{c}{f_1}$$

$$\lambda_c := \frac{c}{f_c}$$

$$\lambda_2 := \frac{c}{f_2}$$

$$\lambda_c = 3.329 \text{ mm}$$

$$\lambda_2 = 2.883 \text{ mm}$$

$$\text{Depth of first slot (maximum): } \frac{\lambda_2}{2} = 1.441 \text{ mm}$$

$$\text{Depth of last slot: } \frac{\lambda_c}{4} = 0.832 \text{ mm}$$

Number of slots per wavelength should be 2.5 or greater at the highest wavelength. The 12-meter 3mm feed (90-116 GHz) had 4 slots per λ at the high end. If we make 3.2 slots per λ at the high end we should be OK and manufacture is less \$\$ for less slots. So:

Pitch(p) is groove width (b) plus ridge width (t) :

$$p := .9 \cdot \text{mm}$$

$$b := .45 \cdot \text{mm}$$

$$t := .45 \cdot \text{mm}$$

Choose a beginning circular waveguide radius (0.125 - inches). This matches commercially available rectangular to circular transitions.

$$\text{Beginning circular radius: } a_0 := 1.59 \cdot \text{mm}$$

Choose a final circular waveguide radius as large as possible but without exciting the TM₁₁ mode in the band:

$$\text{Ending circular radius } a := 1.70 \cdot \text{mm}$$

Waveguide Cutoffs

$$\text{Rectangular } \text{TE}_{10} := \frac{c}{2(.1) \cdot 25.4 \cdot \text{mm}} \quad \text{TE}_{10} = 59.014 \text{ GHz}$$

$$\text{Circular } \text{TE}_{11} := \frac{87.8 \cdot \text{GHz} \cdot \text{mm}}{a} \quad \text{TE}_{11} = 51.647 \text{ GHz}$$

$$\text{Circular } \text{TM}_{01} := \frac{115 \cdot \text{GHz} \cdot \text{mm}}{a} \quad \text{TM}_{01} = 67.647 \text{ GHz}$$

$$\text{Circular } \text{TM}_{11} := \frac{183 \cdot \text{GHz} \cdot \text{mm}}{a} \quad \text{TM}_{11} = 107.647 \text{ GHz}$$

$$\lambda_g := \frac{\lambda_1}{\sqrt{1 - \left(\frac{\text{TE}_{11}}{f1}\right)^2}} \quad \lambda_g = 5.129 \text{ mm} \quad B(\text{zz}) := c1 \cdot c2 \cdot \sec(c2 \cdot \text{zz}) \cdot \tan(c2 \cdot \text{zz})$$

Rule-of-thumb: 5 guide wavelength transition for every factor of 2 change in impedance. However, our change is only 6 percent.

$$5 \cdot \lambda_g \cdot 0.06 = 1.539 \text{ mm}$$

To be safe, and since space is not at a premium, we can use 5 mm, about equal λ_g

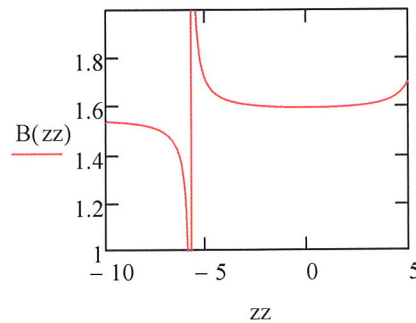
Since the circular waveguide begins in a zero-slope and ends in a 10.9 degree angle, it would be good to fit a curve to this for a smoother transition. This can be done by using:

$$\text{zz}_i := \frac{i}{10} \quad c1 := .028 \quad c2 := .274$$

$$B(\text{zz}) := 1.59 - c1 + c1 \cdot \sec(c2 \cdot \text{zz})$$

$$A(0) =$$

$$A(5) =$$



Make a table of the circular waveguide radius at the beginning of each slot. Assume that the radius increases according to the 10.9 degree horn flare angle and the first slot starts right at the end of the circular waveguide taper section

$$m := 0, 1 \dots 10$$

$$r1_m := 1.70 \cdot \text{mm} + \sin(10.9 \cdot \text{deg}) \cdot (m \cdot p)$$

$r_l =$		0	mm	$\frac{r_l}{\lambda_1} =$		0	$\frac{r_l}{\lambda_c} =$		0	$\frac{r_l}{\lambda_2} =$		0
	0	1.700			0	0.442		0	0.511		0	0.590
	1	1.870			1	0.487		1	0.562		1	0.649
	2	2.040			2	0.531		2	0.613		2	0.708
	3	2.211			3	0.575		3	0.664		3	0.767
	4	2.381			4	0.619		4	0.715		4	0.826
	5	2.551			5	0.664		5	0.766		5	0.885
	6	2.721			6	0.708		6	0.818		6	0.944
	7	2.891			7	0.752		7	0.869		7	1.003
	8	3.061			8	0.797		8	0.920		8	1.062
	9	3.232			9	0.841		9	0.971		9	1.121
	10	3.402			10	0.885		10	1.022		10	1.180

$s_w :=$	1.41	mm	$\frac{s}{\lambda_c} =$		0	$\frac{s}{\lambda_1} =$		0	$\frac{s}{\lambda_2} =$		0
	1.20			0	0.424		0	0.367		0	0.489
	1.06			1	0.361		1	0.312		1	0.416
	.97			2	0.318		2	0.276		2	0.368
	.93			3	0.291		3	0.252		3	0.336
	.90			4	0.279		4	0.242		4	0.323
	.88			5	0.270		5	0.234		5	0.312
	.86			6	0.264		6	0.229		6	0.305
	.84			7	0.258		7	0.224		7	0.298
	.83			8	0.252		8	0.219		8	0.291
				9	0.249		9	0.216		9	0.288

Definitions

$$\text{arcsec} \equiv \frac{\text{deg}}{3600}$$

$$\text{dB} \equiv 1 \quad \mu\text{m} \equiv 10^{-6} \cdot \text{m}$$

$$\text{Tolerance:} \quad \text{TOL} = 1.000 \times 10^{-8}$$

First root of zero order Bessel function of first kind

$$j_{01} \equiv 2.404825577$$

Speed of light

$$c \equiv 299792458 \quad -1$$