CROSS-POLARISATION AND ASTIGMATISM IN MATCHING GROOVES

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Abstract

Circular grooves are frequently cut in dielectric lenses and windows to form antireflection layers. These induce a phase difference between polarisation components, with consequent loss of power into cross-polarisation and a phase error similar to astigmatism. Expressions derived for these losses show that the losses are generally small compared to the insertion losses of an unmatched surface. For several consecutive surfaces the effects may be quite significant.

Introduction

The theory of antireflection layers is well known [1]. For a single matching layer between vacuum and a dielectric with a refractive index n the optimum refractive index for the matching layer is $n_m = \sqrt{n}$ and the optimum thickness is

$$t_{opt} = \frac{l}{4\sqrt{n}} \qquad (1)$$

From microwave to infrared frequencies it is common to practice to machine grooves into a dielectric surface to produce a matching layer with a suitable effective refractive index. If the grooves are narrow compared to a wavelength they constitute an essentially homogeneous dielectric and the effective refractive index can be fixed by appropriate choice of the geometrical parameters. The effective dielectric constant depends on the ratio a between the pitch of the grooves, *p*, and the groove width, *g*.

Although the grooved region is homogeneous it is not isotropic and has a refractive index parallel to the grooves given by

$$n^2_p = a + (1 - a)n^2 \qquad (2)$$

while normal to the grooves it is

$$n^2_n = \frac{n^2}{1 - a + a\,n^2} \qquad (3)$$

Typically rectangular or triangular profiles are used as shown in Fig. 1. In the rectangular grooves the refractive index is constant, while triangular grooves have an effective index varying continuously with depth (*i.e.*, a goes from 0 to 1). More accurate expressions for the effective refractive indices which apply even for groove pitches approaching a wavelength are given by Bräuer and Bryngdahl [2].

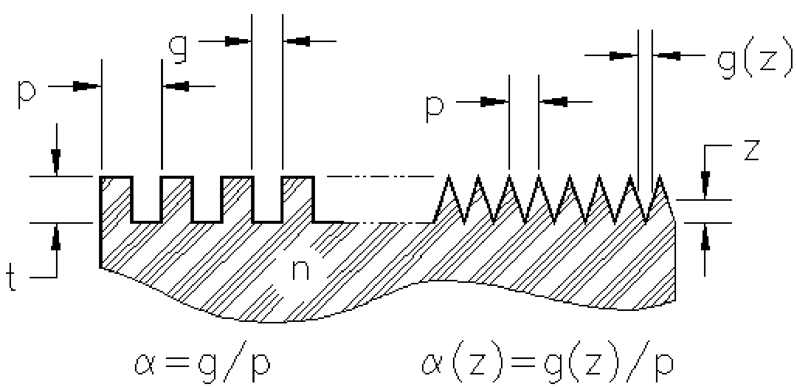

Fig. 1 Geometry and nomenclature used in the text.

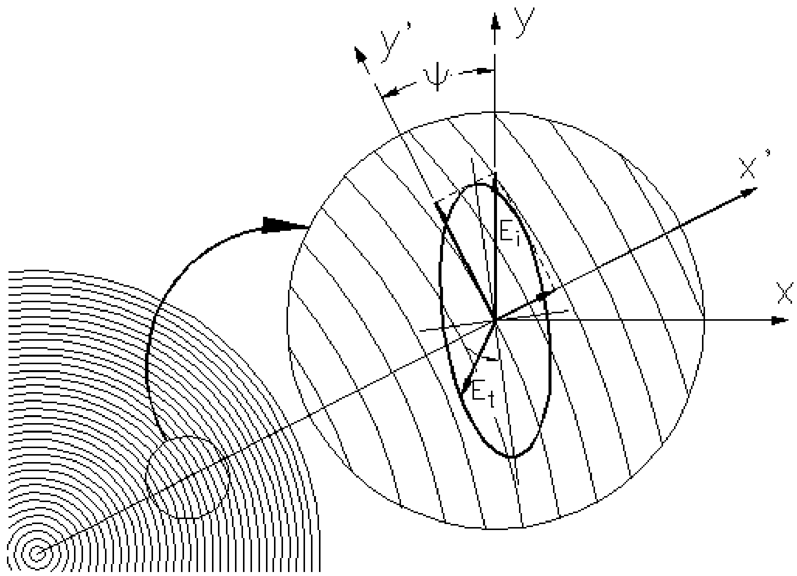

Fig. 2 Notation for y-polarised field incident on circular grooves.

Cross-Polarisation

If the grooves are parallel or normal to the electric field there is no change of the polarisation state of the field. However, grooves are often machined in circular paths, on lenses for example, so that the direction of polarisation varies relative to the groove (Fig. 2). In such a case the electrical pathlengths for the two components of the field are different, as is seen from 2 and 3, and a linearly polarised wave becomes elliptically polarised.

Consider a y-polarised electric field $\boldsymbol{E}_i$ incident on grooves that are locally at an angle Ψ (Fig. 2) relative to the plane of polarisation. In the coordinates of the incident field this may be written

$$E_i = \begin{pmatrix} 0 \\ E_i \end{pmatrix} \quad (4)$$

In the coordinates of the grooves denoted by the prime this becomes

$$E_{i'} = \begin{pmatrix} E_i \sin(\Psi) \\ E_i \cos(\Psi) \end{pmatrix} \quad (5)$$

The grooves introduce a differential phase delay d between the two components, so that the transmitted field, E_t, is now elliptically polarised. In the groove coordinate frame:

$$E_{t'} = \begin{pmatrix} E_i \sin(\Psi) \\ E_i \cos(\Psi)\, e^{i d} \end{pmatrix} \quad (6)$$

Converting back to the coordinate frame of the incident field gives

$$E_t = \begin{pmatrix} E_i \cos(\Psi) \sin(\Psi) \left(1 - e^{i d}\right) \\ E_i \left(\sin^2(\Psi) + \cos(\Psi)^2 e^{i d}\right) \end{pmatrix} \quad (7)$$

The first element is the cross-polarised component of the field for which the power is

$$P_x = \frac{E_i^2}{2} \sin(2\Psi)^2 (1 - \cos(d)) \quad (8)$$

If a circularly symmetric beam, *E(r)*, is incident on a set of circular grooves the total power lost into the cross-polar is found by integration:

$$P_{x,tot} = \frac{\int_0^a \int_0^{2p} \frac{1}{2} E(r) \sin^2(2\Psi)(1 - \cos(d))\, d\Psi dr}{\int_0^a \int_0^{2p} E(r)\, d\Psi dr} \quad (9)$$

which simplifies to

$$P_{x,tot} = \tfrac{1}{4}(1 - \cos(d)) \quad (10)$$

or for small values of d

$$P_{x,tot} \approx \tfrac{1}{8} d^2 \quad (11)$$

Note that the axes of the vibration ellipse are not aligned with either coordinate system. The field component along the minor axis has a power somewhat less than given by 8.

d is found from the difference between the refractive indices in the two principal directions (parallel and normal to the grooves). If the widths of the grooves are a function of the depth then the effective refractive indices will also be a function of the depth, $n_p(z)$ and $n_n(z)$. The total phase delay is then

$$d = \frac{2p}{l} \int_0^d (n_n(z) - n_p(z))\, dr \quad (12)$$

where d is the groove depth. For a rectangular groove with depth d the phase difference is

$$d = 2pd\, \frac{n_p - n_n}{l} \quad (13)$$

$$d = \frac{4\,p\,d}{3\,l} \frac{n^2 - 2n + 1}{(n+1)} \quad (14)$$

while for triangular grooves the integral yields

Clearly even an ungrooved lens can generate cross-polarisation because of the differing reflection coefficients for the two polarisations. This is more difficult to calculate and depends on the lens shape and the dielectric. Numerical results for several cases typically gave about -40 dB cross-polar, considerably less than for the grooves. Of course, the reflection is much higher, typically several percent.

Astigmatism

Another effect of the difference in the orthogonal refractive indices is the variation of the phase of the copolar component of the field. An approximation to the phase, f, of the y-component of the field given in 7 is

$$f = d\cos^2(\Psi) \tag{15}$$

which is similar to an astigmatic aberration, though without the usual radial dependence. From this the gain reduction may be found [3]

$$\frac{\Delta G}{G} = \overline{f^2} - \overline{f}^2 = \tfrac{1}{8} d^2 \tag{16}$$

which is coincidentally the same as the cross-polarisation loss.

Discussion

As an example, take a dielectric matching layer cut into PTFE for which $n = 1.44$. For rectangular grooves it is reasonable to take $a = 0.5$ as a compromise between the ideal values for the two polarisations. For a design frequency of 230 GHz the depth of the groove is $d = 0.272$ mm. Without grooves the power reflection per surface is about 3 %. With the grooves the reflection is reduced as shown in Table I. There is no frequency at which the reflection is zero for both polarisations so that the minimum total reflection is about 0.1 %.

For m surfaces the cross-polar loss increases by m^2, unless the cross-polarised component is removed between the surfaces with grids, for example. In that case the loss increases as m. The phase error losses of a series of ungrooved surface are not reduced by the grids and will increase as m^2.

Triangular grooves are generally deeper than rectangular ones. For a depth of one wavelength the losses into cross-polar and astigmatism are each about 1 %.

Clearly the losses associated with the grooves are less than the losses of the unmatched interface in most cases, though deep triangular grooves should be avoided. The cross-polar should be well terminated, but generally the loss is too small to justify a cryogenic termination unless it is anyway convenient to do so.

Table I : Losses due to circular grooves for a single matched surface.

Loss, %		Frequency 200 GHz	230 GHz	260 GHz
No grooves:	**Reflection**	3.2	3.2	3.2
Rectangular grooves:	**Reflection**	0.25	0.11	0.25
	X-pol	0.10	0.13	0.17
	Phase	0.10	0.13	0.17
	Total	0.45	0.37	0.59

References

1. M. Born and E. Wolf: *Principles of Optics*, 6th ed., Oxford: Pergamon, 1980.

2. R. Bräuer and O. Bryngdahl, "Design of antireflection gratings with approximate and rigorous methods," *Appl. Opt.*, vol. 33, no. 34, pp. 7875-7882, Dec. 1994.

3. J. Ruze: "Antenna tolerance theory—A Review," *Proc. IEEE*, Vol. 54, pp. 633-640, April 1966.