Atacama
Large
Millimeter
Array

Polarization Calibration Steps

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Contents

1 Goals

Polarization calibration essentially determines the cross-talk between the two measured orthogonal source polarizations (see Appendix A.1 for details). The cross-talk terms are generated in the feeds, by the antenna structure and blockages, and from the receivers. Assuming polarization types X and Y (R and L circular feeds can replace X and Y linear feeds for most of this discussion), the cross-talk can be parameterized for each antenna at a specified frequency, time, and at the beam center by:

d_X = amount of Y polarization signal in X polarization channel.

d_Y = amount of X polarization signal in Y polarization channel.

Both of these terms are complex numbers giving the amount and relative phase of the leakage signal. Other parameters which tie the phases between all of the X and Y polarization channels are:

ϕ_{XY} = phase difference between all X and Y polarization channels

δ_{XY} = delay difference between all X and Y polarization channels

It is expected that the d_X and d_Y terms will be about 2% for the lower frequencies, but perhaps up to 10% at the highest frequencies. Calibration accuracy, based on VLA experience, should be relatively easy to the 0.5% level per antenna at the lower frequencies. Thus, even at this level, polarization image artifacts should be less than 0.1% of the peak on the total intensity image since the individual antenna cross-talks will cancel each other.

2 Calibration steps

2.1 Apriori Calibrations

The calibration of the delay difference between the two total intensity channels, X and Y, to determine δ_{XY}, will probably be done as an ALMA service calibration. This difference will only change when there are physical changes made in the X and Y receiving systems. This crossed-hand delay offset can be determined by observing a strongly polarized source (its polarization percentage and angle need not be known), with strong enough polarized signal above the d term level ($> 5\%$). A relatively accurate bandpass calibration may also needed before determining the crossed-hand delay.

2.2 Normal Amplitude and Phase Calibrations

The full-Stokes calibration of interferometric data is generally done in two steps: (1) calibration the total intensity data; (2) calibrate the cross-polarization data using the total intensity callibration. For most observations with relatively small degrees of polarization, second order terms are $< 1\%$. The CASA package may, in fact, have the ability to calibrate all Stokes parameters in one step.

The calibration of the amplitude, phase, bandpass, *etc.*, calibrations for the X and Y polarization channels (again for R and L), as described in other calibration documents.

2.3 Crossed-Polarization Terms and Polarization Phase Difference

These calibrations should be done in interferometric mode, and the measurements of the d_X and d_Y can often be obtained from the phase referencing calibrator observations, used to determine the residual amplitude and phase during the experiment. However, determination of accurate cross-polatization terms requires observations over several hours in order to obtain sufficient parallactic angle coverage to separate out the cross-polarization terms and the calibrator polarization. If the calibrator polarization is known (or unpolarized), then the crossed-polarization terms are easily deetermine.

The d_X and d_Y are relatively stable over time, particularly over an experiment of several hours, since they depend on imperfections in the polarization purity of the feeds. Barring receiver and configuration changes, sufficiently accurate determination of d_X and d_Y could be provided during nominal array calibrations and be accurate enough for most target observations with no additional polarization calibrations.

There will probably be a significant change of the polarized cross-talk over the observed frequency in an experiment. If the changes are less than a few percent, then the average cross-talk determined from all spectral channels should be sufficiently accurate. However, for high dynamic range polarization imaging especially of different spectral features, the above cross-talk calibration may have to be done at many frequency points in the spectral range.

The intrinsic polarization angle, ϕ_{XY}, even after calibration of the total intensity and leakage terms, is generally unknown. This is essentially the phase difference between all of the X and all of the Y channels. It can be obtained by observing a strong source known polarization angle and changing the relative phase of the two total intensity channels to get the known angle for the source. It is similar to the absolute flux density calibration of the data in that it is one phase offset between all X and all Y channels. This calibration is describe in more detail below.

An exhaustive discussion of polarimetric calibration in the measurement equation approach is contained in [2]. The following summarizes the steps necessary to carry out polarimetric calibration using the [2] formalism:

1. Set up standard interferometric observing in desired band, frequency setting, and correlator mode.
2. Observe a bright calibrator (with sufficient signal-to-noise to solve for needed quantities). Source need only be bright enough in total intensity (Stokes I) over coherent solution intervals (though significant measureable known polarization allows extra constraints). Source should be compact (preferably point-like). Choice of calibration source determines the sequence of observations:
   (a) *Unpolarized calibrator* — need only observe single scan. Can determine d_X and d_Y up to a single complex number (equivalent to offsetting all d's or setting one to zero) that can be absorbed into gains, plus an unknown ϕ_{XY}.
   (b) *Polarized calibrator, polarization known* — also need only observe a single scan. Can determine d_X and d_Y up to a single unknown amplitude factor that can be absorbed into gains. The phase ϕ_{XY} is fixed by the calibration (see below) to the extent that the source has known and measurable linear polarization.
   (c) *Polarized calibrator, polarization unknown* — need to observe calibrator in at least 3 scans spanning a range of at least 90° in parallactic angle. In general, this means the calibration source needs to be at a declination within ±20° of the observatory latitude. Can determine d_X and d_Y up to a single complex number that can be absorbed into gains and ϕ_{XY}, as well as the intrinsic source polarization (up to the unknown phase which for linear polarizations X,Y is an ambiguity in the mixture of Stokes U and V).

3. *Polarization Phase Difference*: Even with accurate determination of the d_X and d_Y terms, the calibration of the intrinsic angle of the electric vector of the polarization is not determined. This requires an observation of a polarized source with *known* position angle to, in effect, determine the phase difference between the X phases and the Y phases. In case (a) above, an additional observation of a polarized source must be included. In case (b) this scheme will determine both d_X and d_Y with the one source. In case (c), although the degree of polarization of the source is determine with good parallactic angle coverage, the orientation of the electric vector is still unkown. Hence, an additional observation of a polarize source of *known* position angle must also be included. This calibrator can be somewhat resolved since the polarization angle calibration the single X versus Y rotation that applies to all baselines. Hence an accurate measurement of this angle at the shorter baselines may be sufficiently accurate.
4. The necessity to know the true polarization angle of relatively bright, polarized ($> 5\%$) quasars at ALMA frequencies is unclear. Most quasars that have a high degree are very variable and most thermal sources have very little polarizations. Hence, the search for a set of good polarization calibrators over the sky should be incorporated into the search for a set of good primary flux density calibrators, and incorporated with the monitoring of the flux density of secondary quasars. Currently, there is a 23-hour VLA observation every two weeks for this type of monitoring. The present specification of an accuracy of $6°$ for the intrinsic angle of polarization, however, is relatively benign and it should be relatively straight-forward finding calibrator polarizations which are accurate to this precision.
5. After observation and amplitude and gain/bandpass calibration, solve for d_X and d_Y using the measurement equation.
6. Store results for transfer to calibration source or use by other observing programs using same setup (band and frequency channels).
7. During further calibration and/or imaging steps, apply D matrix including d-terms.
8. Appropriate software in casa or other packages will determine the values of d_X, d_Y for each antenna/quadrant (four of them), and the global ϕ_{XY} term. This determination will probably be done in the first pipelining of the data, after the normal total power calibrations, and applied to all of the relevant data.

2.4 Polarization Leakage Beam

The above calibrations are associated with the polarization properties at the center of beam for a particular observing frequency. The cross-terms are, however, a property of frequency (discussed above) and location in the primary beam. Thus, similar measurements should be made in interferometric mode to determine the $d_X(\boldsymbol{\theta})$ and $d_Y(\boldsymbol{\theta})$at over the primary beam. Since these antenna properties should be stable with time (barring changes in the feed and antenna structure), this beam mapping should be made in conjunction with the beam properties of the total intensity. Certainly, interferometric beam making will deal with the crossed-terms, but it is unclear if holographic measurement of the beam (see Primary Beam calibration steps) can also be used for the cross-term beam properties.

Thus, the ALMA system must provide these beam properties in total intensity and crossed polarization for check-out of the antennas and for use in detailed imaging of sources which fill the primary beam. It is unlikely that single experiments will have the necessary time to determine the instrumental fuctions.

2.5 Quarter Wave Plate (QWP)

The need for large spectral ranges for each ALMA band requires that the feeds of all ALMA bands will be dual-linearly polarized. For radio sources that are linearly polarized, circular-polarized feeds are easier to calibrate, but calibration of the linear-feeds should easily reach the desired polarization purity for most ALMA observations.

However, for Band 7 (345 GHz) for which the feeds are on the optical axis, the possibility of high-quallity polarization imaging is possible. Hence, all antennas in the band will be equipped with a quarter wave plate (QWP) which will be injectable within the receiver beam in order to produce dual circular polarized data streams. The specifications of these QWPs are as follows:

— *Center Frequency and Range:* 340 GHz; 329–351 GHz
— *Combined Absorptive and Reflective Losses:* < 0.25 dB
— *Induced Cross-Polar Component:* $< 3\%$

This option will permit polarization imaging of sources to a level of about 0.1% purity for this particular frequency range. The other bands are already somewhat impeded in spectral purity to 0.5% because the fields are somewhat off the optical axis, so that very deep polarization imaging would not be possible, even with addition of the QWP to derive circular polarization data streams.

3 Further Issues

The calibration steps outlined above should cover most cases that the baseline ALMA will encounter. However, there are still a number of issues that have not covered here, including:

— Single-dish determination of polarization parameters
— Time dependence of **D** and beam changes.
— **D** determination using extended sources
— Optimal factorization of **D** into more physical terms
— Application of the calibration parameters during processing

3.1 Single Dish Polarimetry

Total power polarization calibration can be done by placing the total power antennas in interferometric arrays and calibrating their polarization as described above. Since it is expected that the total power telescopes will occasionally enter into array observations for general calibrations during a total power mosaic or on-the-fly, a few observations of appropriate polarizatin calibrator sources are suggested. For example a relatively short observation of an unpolarized calibrator and a known polarized calibrator (perhaps several minutes integration on each) will be sufficient to determine the total power antenna polarization interferometric response. However, the difference between the total power antenna response as used as an interferometer and as used as a single dish may not be negligible. Further understanding will be obtained during commissioning.

3.2 Time Dependence of d_X and d_Y

How constant are d_X and d_Y? It is generally assumed that the leakage signals between polarization channels is relatively constant over periods of days to weeks, with no changes to an antenna (configuration change or major maintenance). This means that it may be possible to determine d_X and d_Y along with some of the other system calibrations (perhaps including this calibration as part of the antenna location observations) with an accuracy of perhaps 1%. This may be sufficiently accurate for low to medium SNR observations. In addition, it is expected that the differential polarization changes over each antenna beam will be constant with time unless stuctural changes are made.

3.3 Frequency Dependence of d_X and d_Y

The leakage term d_X and d_Y may be a function of frequency setting at the level of a few percent. These changes are main caused by low level reflections between the subreflector and the antenna surface which change the polarization leakage by a few percent with a character frequency of a few MHz. This effect should be investigated during the commissioning phase. This reflection may also be a function of source elevation and thus produce an effective change of d_X and d_Y. This change is generally less than 0.5%, but again should be checked during commissioning. If these terms are larger than a few percent, then the d_X and d_Y determinations as a function of frequency (easy) and time (harder) for polarization observations with $< 0.5\%$ purity may have to be included in the observing program.

3.4 Time Dependence of ϕ_{XY}

A time variable ϕ_{XY} (phase difference between the X and Y receivers) causes a major problem in the polarization determination. Generally, the choice of reference antenna in the determination of the polarization parameters (and also most of the other interferometrically determined parameters) is somewhat critical. One important property is the phase difference between the X and Y channels for the antennas, and the one with the smallest change (over periods of several hours) should be chosen as the reference antenna. Alternatively, one can use a reference antenna which is equal to the sum of all antennas to obtain a more stable average phase difference between polarizations. Another consideration is to choose a reference antenna near the array center so that the antenna-based troposphere phase fluctuations from other antennas are somewhat minimized. For precision polarimetry, a variable phiXY over the experiment period may have to be determined. This is not a major problem to measure but is not yet incorporated in any of the software.

If time variability is a problem with ALMA, then a relatively strongly polarized source should be observed several times over the experiment to measure the phase change.

3.5 Beam Dependence

The polarization calibrations of d_X and d_Y, described above, determine the parameters at the *center of the primary beam*. They are known to vary by several percent or more at the 10% power level of the primary beam. Hence the polarization calibration must be determined at many points in the primary beam in order to determine the appropriate corrections to extended sources obtained from single pointing, mosaicing and even on-the-fly imaging, although the polarization SNR here may be relatively low. The recommendation is to determine the complete stokes representation of the primary beam pattern $X(\boldsymbol{\theta})$, $Y(\boldsymbol{\theta})$, $d_X(\boldsymbol{\theta})$, $d_Y(\boldsymbol{\theta})$, where $\boldsymbol{\theta}$ is the two-dimensional angle displacement from the beam center, when detailed primary beam mapping observations are made. These observations should be done at commissioning and also at periodic intervals of a few months or so.

3.6 Polarization Self Calibration

It is possible that polarization self-calibration techniques will be developed in order to determine the d_X and d_Y terms on the bright source much more accurately than that obtained with the above calibration techniques.

However, the primary beam dependence of d_X and d_Y will probably never by obtainable with self-calibration technique. Also, the intrinsic polarization angle can only be determined using an external calibrator with known polarization angle at the time of the observations.

References

[1] Hamaker, Bregman, & Sault 1996, A&AS, 117, 137

[2] Sault, Hamaker, & Bregman 1996, A&AS, 117, 149

[3] ALMA Calibration Plan (2004-10-07 Draft), SCID-90.03.00.00-001-A-PLA

A The Measurement Equation for Polarimetry

A.1 Polarization Interferometry

The complete description of the full calibration of interferometric data is given below in the measurement equation (*c.f.* [1]). The polarization calibration steps outlined in this document are concerned with determining the elements of the leakage **D** and the relative phase between the two polarization gains in **G**.

We assume that separate calibration procedures have yielded:

— relative gains g_X and g_Y (up to the phase difference ϕ_{XY}) inside **G** (see Equation 10),

— measurements of the antenna voltage patterns E_X and E_Y through holography or beam mapping.

We must then determine

— polarization leakage factors d_X, d_Y

— polarization leakage "beams" δ_X, δ_Y.

— phase difference between polarizations ϕ_{XY}.

Equation 9 shows the usual separation of the leakage term into the beam center leakage d_X and d_Y, and the leakage beam terms, δ_X and δ_Y. The intrinsic polarization angle term is show by ϕ_{XY} in Equation 10. The initial non-polarization calibrations that should be made before polarization calibrations are: P = parallactic angle of feed G = Complex gain of each antenna as a function of time and frequency (bandpass).

The fundamental quantity that must be measured and eventually applied for each antenna is the total matrix relating the true polarization of the radiation incident on the antenna aperture and the measured polarization as a function of direction off the nominal pointing direction

$$\hat{\boldsymbol{e}}_k(\boldsymbol{\theta},\nu,t) = \mathbf{J}_k(\boldsymbol{\theta},\nu,t)\,\boldsymbol{e}_k(\boldsymbol{\theta},\nu,t) \tag{1}$$

where the Jones matrix **J** ([1]) relates the input *voltage* e to an output voltage $\hat{e}$ for antenna k as a function of the direction $\boldsymbol{\theta}$ at frequency ν and time t. The voltage vectors $\boldsymbol{e}$ have a component for each of the orthogonal polarizations received by the system. For most ALMA bands and setups, we will be measuring two linear polarizations X,Y though there may be a widget in one band to measure circular polarizations R,L (and an eventual Band 1 might also be RL). For the following, we will therefore assume crossed linears.

Expanding the measurement equation gives

$$\begin{pmatrix} \hat{e}_{kX}(\boldsymbol{\theta},\nu,t) \\ \hat{e}_{kY}(\boldsymbol{\theta},\nu,t) \end{pmatrix} = \begin{pmatrix} J_{kXX}(\boldsymbol{\theta},\nu,t) & J_{kXY}(\boldsymbol{\theta},\nu,t) \\ J_{kYX}(\boldsymbol{\theta},\nu,t) & J_{kYY}(\boldsymbol{\theta},\nu,t) \end{pmatrix} \begin{pmatrix} e_{kX}(\boldsymbol{\theta},\nu,t) \\ e_{kY}(\boldsymbol{\theta},\nu,t) \end{pmatrix}. \tag{2}$$

This is traditionally (and practically) broken into several separate stages, each characterized by a different Jones matrix. These are strung together in what is called the *measurement equation*. The polarization calibration follows the signal path from the dish through to the correlator or total power detector (for interferometry or single dish respectively). Thus, we write, in mulitiplicative order,

$$\mathbf{J}_k(\boldsymbol{\theta},\nu,t) = \mathbf{G}_k(\nu,t)\,\mathbf{D}_k(\boldsymbol{\theta},\nu)\,\mathbf{E}_k(\boldsymbol{\theta},\nu)\,\mathbf{P}_k(t) \tag{3}$$

where we are for the moment assuming that only the (bandpass) gain G is time-dependent, while the polarization leakage D and antenna voltage pattern E vary on substantially longer timescales (if at all).

The parallactic angle matrix $\mathbf{P}$ converts the sky coordinate system for X,Y into the antenna coordinate system for the receivers and is calculable. The rotation to be done to convert the X,Y axes of the altitude-azimuth mounted receivers to the actual on-sky coordinate systems is given by

$$\mathbf{P}_k = \begin{pmatrix} \cos\chi_k(t) & \sin\chi_k(t) \\ \sin\chi_k(t) & \cos\chi_k(t) \end{pmatrix} \tag{4}$$

for parallactic angle χ which rotates on the sky as a function of time t. For ALMA, all antennas can be defined to have the same parallactic angle χ at any given time t (not true for VLBI), but also note that if the various receivers have their feeds or polarizing grids oriented at different angles with respect to the elevation axis, then this is best dealt with by adjusting the $\mathbf{P}$ matrix, otherwise this will show up in other terms (see below). The matrix P changes if one uses a circular polarization basis, to

$$\mathbf{P}_k = \begin{pmatrix} e^{i\chi_k(t)} & 0 \\ 0 & e^{-i\chi_k(t)} \end{pmatrix}. \tag{5}$$

The individual G, D, and E Jones matrices are chosen to factorize into the following gain

$$\mathbf{G}_k(\nu,t) = \begin{pmatrix} G_{kX}(\nu,t) & 0 \\ 0 & G_{kY}(\nu,t) \end{pmatrix}, \tag{6}$$

leakage

$$\mathbf{D}_k(\boldsymbol{\theta},\nu) = \begin{pmatrix} 1 & D_{kX}(\boldsymbol{\theta},\nu) \\ D_{kY}(\boldsymbol{\theta},\nu) & 1 \end{pmatrix}, \tag{7}$$

and voltage pattern

$$\mathbf{E}_k(\boldsymbol{\theta},\nu) = \begin{pmatrix} E_{kX}(\boldsymbol{\theta},\nu) & 0 \\ 0 & E_{kY}(\boldsymbol{\theta},\nu) \end{pmatrix} \tag{8}$$

terms which contain the 6 quantities that must be solved for either outside or during the observations.

We further factorize D into direction independent and dependent terms

$$\mathbf{D}_k(\boldsymbol{\theta},\nu) = \begin{pmatrix} 1 & d_{kX}(\nu) + \delta D_{kX}(\boldsymbol{\theta},\nu) \\ d_{kY}(\nu) + \delta D_{kY}(\boldsymbol{\theta},\nu) & 1 \end{pmatrix} \tag{9}$$

in order to allow simplification for determination of the d through simple on-axis calibration and leaving the leakage “beam” δD as an increment to the independent term. Finally, we factorize the gain $\mathbf{G}$

$$\mathbf{G}_k(\nu,t) = \begin{pmatrix} g_{kX}(\nu,t) & 0 \\ 0 & g_{kY}(\nu,t)\, e^{i\phi_{kXY}(\nu,t)} \end{pmatrix} \tag{10}$$

to take up the ambiguities tying the polarization channels together during separate calibration of the other quantities, up to the phase difference $\phi_{kXY}(\nu,t)$ between the two polarizations.

Once the observed electric fields have been related to the on-sky X and Y polarization of the incoming wavefronts through Equation 3, we can cross-correlate the signals from the two polarizations on the two antennas of a given baseline to form the four correlation products. These correlation products for each "visibility" $V_{kk'}$ between the pair of antennas k, k' are given by the *outer product*

$$\hat{\boldsymbol{V}}_{kk'} = \langle \hat{\mathbf{e}}_k \otimes \hat{\mathbf{e}}^*_{k'} \rangle = \mathbf{J}_k \otimes \mathbf{J}^*_{k'} \boldsymbol{V}_{kk'} \tag{11}$$

where

$$\hat{\boldsymbol{V}}_{kk'} = \left\langle \begin{pmatrix} e_{kX} \\ e_{kY} \end{pmatrix} \otimes \begin{pmatrix} e^*_{k'X} \\ e^*_{k'Y} \end{pmatrix} \right\rangle = \begin{pmatrix} V_{kk'XX} \\ V_{kk'XY} \\ V_{kk'YX} \\ V_{kk'YY} \end{pmatrix}. \tag{12}$$

In the celestial frame, the polarization products are related to the Stokes parameters for the source by

$$\begin{pmatrix} V_{XX} \\ V_{XY} \\ V_{YX} \\ V_{YY} \end{pmatrix} = \begin{pmatrix} I + Q \\ U + i * V \\ U - i * V \\ I - Q \end{pmatrix}. \tag{13}$$

for crossed linears, and

$$\begin{pmatrix} V_{RR} \\ V_{RL} \\ V_{LR} \\ V_{LL} \end{pmatrix} = \begin{pmatrix} I + V \\ Q + i * U \\ Q - i * U \\ I - V \end{pmatrix}. \tag{14}$$

for a circular polarization basis.