To: ALMA IPTs
From: R. Sramek
Subject: Phase errors arising from baseline errors
Date: 17-Oct-2005

The predicted geometric phase when pointed at source A is given by:

$$\Phi_A = \vec{B} \bullet \vec{S}_A(t_1) \qquad (1)$$

where:
$\vec{B}$ is the baseline vector between two antennas and
$\vec{S}_A(t_1)$ is a unit vector pointing in the direction of source A at time t_1.

Assuming that the source is unresolved, the measured interferometer phase, i.e. the output of the correlator, arising from errors in the assumed baseline or the source position is given by:

$$\delta\Phi_A(t_1) = \delta\vec{B}(t_1) \bullet \vec{S}_A(t_1) + \vec{B} \bullet \delta\vec{S}_A(t_1) + \phi_A(t_1) \qquad (2)$$

where:
$\phi_A(t_1)$ is the instrumental phase arising in the electronics system at time t_1 while pointing at radio source A.

A similar expression can be written for calibrator source C

$$\delta\Phi_C(t_2) = \delta\vec{B}(t_2) \bullet \vec{S}_C(t_2) + \vec{B} \bullet \delta\vec{S}_C(t_2) + \phi_C(t_2) \qquad (3)$$

Differencing the phase of the target source and calibrator, the calibrated interferometer phase is given by:

$$\Delta\Phi_{AC} = \delta\Phi_A(t_1) - \delta\Phi_C(t_2)$$

$$\Delta\Phi_{AC} = [\delta\vec{B}(t_1) \bullet \vec{S}_A(t_1) - \delta\vec{B}(t_2) \bullet \vec{S}_C(t_2)] + \vec{B} \bullet [\delta\vec{S}_A(t_1) - \delta\vec{S}_C(t_2)] + [\phi_A(t_1) - \phi_C(t_2)] \qquad (4)$$

For simplicity, assume that both the target source and calibrator have accurately known positions and $\delta\vec{S}_A = \delta\vec{S}_C = 0$. The residual calibrated instrumental phase,

$\Delta\Phi_I = \phi_A(t_1) - \phi_C(t_2)$,

can be made very small if the instrumental phase changes slowly on time scales of $t_1 - t_2$ and does not depend on where the array is pointing. Faster instrumental phase changes would add a random phase noise to the source visibilities and create errors in the resulting images. Far more serious would be an instrumental error that depends on array pointing (e.g. phase errors arising in the cable wraps that are not corrected by the LLC); this could add a constant phase offset that would not average down over time with many observations.

Rewriting Equation (4),

$$\Delta\Phi_{AC} = [\delta\vec{B}(t_1) \bullet \vec{S}_A(t_1) - \delta\vec{B}(t_2) \bullet \vec{S}_C(t_2)] + \Delta\Phi_I \qquad (5)$$

Now $\delta\vec{B}$ is the error in the baseline vector. It could arise from errors in determining the baseline vector at the time of the baseline calibration observation, or it could arise from movement of the antenna after the baseline calibration. If $\delta\vec{B}$ does not change on the time scale of $t_1 - t_2$, Eq (5) can be simplified,

$$\Delta\Phi_{AC} = \delta\vec{B} \bullet [\vec{S}_A - \vec{S}_C] + \Delta\Phi_I$$
$$\Delta\Phi_{AC} = \delta\vec{B} \bullet \Delta\vec{S}_{AC} + \Delta\Phi_I \qquad (6)$$

where $\Delta\vec{S}_{AC}$ is the vector separation of the source and calibrator on the sky. All quantities in Eq (6) are taken at a time midway between t_1 and t_2. The fact that $\Delta\vec{S}_{AC}$ is much smaller than a unit vector is why the effects of baseline errors are reduced proportional to the angular separation between source and calibrator. However, changes in $\delta\vec{B}$ on time scales greater than $t_1 - t_2$ are not removed by the calibration process. Any error since the last baseline calibration remains. Baseline errors are similar to instrumental errors that repeat with array pointing position.

Changes in $\delta\vec{B}$ on time scales shorter than $t_1 - t_2$ are also not removed by the calibration process; Eq (5) should be used to evaluate these errors. If this rapid change in baseline vector is random, the baseline error will add a noise like term and average down.